Sergiy V. Kryvoruchko

EVALUATION AND FORECASTING OF STAFF PERFORMANCE:

From a Common to a Scientific Approach — Definition and Comparative Analysis of Deterministic and Stochastic Approaches

Dedicated to all
those striving to the Truth

About the Author

Mr. Sergiy Kryvoruchko, Ph.D. in Technical Sciences, Associate Professor, Honored Innovator of Ukraine.

Expert in System Analysis, Business Administration, Business Economics, and HR Management.

Education: Technical (Radio Engineering), Economic (Business), and Diplomatic (Diplomatic and Consular Service).

Work experience: Science and Higher Education (Vice-Chair of the University Department), State Expertise and Science Council (Academic Secretary), State Military Analytical Foreign Affairs Center (Head of the Direction), Military and Diplomatic Activity at the Embassies of Ukraine in the USA and Canada (First Secretary). Over 20 years of entrepreneurship experience (Chief Executive Officer, President).

Founder and owner of a group of successful companies.

Author of the books “Success in Business: From Zero to Millions” and “Key Axioms of Business: The Basic Rules of Starting and Developing a Successful Business”. Author of numerous articles, including on staff recruitment, evaluation and forecasting.

Sergiy V. Kryvoruchko

EVALUATION AND FORECASTING OF STAFF PERFORMANCE:

From a Common to a Scientific Approach—Definition and Comparative Analysis of Deterministic and Stochastic Approaches

KYIV–TORONTO

lat&k
publisher

2021

UDC 65.01+658.8
K82

Sergiy V. **Kryvoruchko**

K82 Evaluation and Forecasting of Staff Performance: From a Common to a Scientific Approach—Definition and Comparative Analysis of Deterministic and Stochastic Approaches. KYIV: LAT & K, 2021.—72 p.
ISBN 978-617-7824-43-4

The book provides a detailed comparative analysis of deterministic and stochastic approaches to staff performance evaluation and forecasting by indicators of informativeness, accuracy, simplicity, cost, and period for measuring the degree of performance (effectiveness, productivity) of specialists working in the same and different conditions, in the same and in different activity areas.

This book formalizes the basic understandings and concepts of evaluating and forecasting staff performance, analyzes in detail the main groups of quantitative and probabilistic indicators for personnel performance evaluation and forecasting, and formulates the minimum required sample sizes of measured values that should ensure the necessary accuracy of the deterministic and stochastic indicators of staff performance evaluation and forecasting.

This research analyzes the different aspects, including the accuracy, time, and volatile benefits of daily staff productivity measurement; presents the results of practical application of a combined set of stochastic personnel evaluation and forecasting indicators in various companies operating in different market segments; substantiates the feasibility of stochastic approaches to evaluating and forecasting staff performance; and determines recommendations for the practical use of probabilistic-mathematical approaches in evaluating and forecasting staff performance.

This book is useful for top managers in government, commercial institutions, and companies, HR managers, research and teaching staff, analysts, and students—in short, to everyone interested in reasonable evaluation and forecasting of staff performance.

UDC 65.01+658.8

ISBN 978-617-7824-43-4

Table of Contents

Introduction

Staff performance measurement (evaluation and forecasting or assessment and prediction) in various enterprises and organizations is extremely relevant in the modern world, both for everyday practice and in its theoretical research aspects.

Today, any government, nonprofit, or commercial company (organization, enterprise, etc.) should measure its human resources productivity. Many leaders are now arguing that the measurement of employee performance should be the primary focus of metrics because human capital (employees) is the most critical asset to achieving an organization's mission [1]. Failure to do this can turn out to be extremely costly and sometimes deadly for modern organizations. The growing focus on measuring human resource productivity, which still lags far behind other functions of employer entities, is due to three factors: the significant impact of high-performance human resource (HR) management systems, the implications of poorly performing HR management, and soaring HR operating expenses [1].

This discussion proceeds from the fact that *performance measurement* [2], which is the process of collecting, analyzing, and/or reporting information regarding the performance of an individual, group, organization, system, or component, is a system [3] that enables informed decisions to be made and actions to be taken because it quantifies the efficiency and effectiveness of past actions through the acquisition, collation, sorting, analysis, and interpretation of appropriate data. This is achieved by obtaining, collecting, sorting, analyzing, and interpreting relevant data about the measured object. Based on this, it is extremely important to choose effective ways to measure certain performance parameters (indicators) of the appropriate measured objects [1, 4], including using key performance indicators [5, 6]. This, in turn, should simultaneously require clear structuring of these ways of measuring performance parameters and formalization of the relevant set of staff performance indicators (as the object of measurement). This should involve mandatory use of opportune approaches at maximum efficiency to process the measured performance values for staff, including officials, professionals, specialists, employees, and workers.

Performance will be understood as [7] a person's achievement, [8] the degree to which a feat is being or has been accomplished, or [9] how well or badly a person does something. In turn, an *evaluation* will be understood as [7] assessing, appraising, finding, or stating the number or amount of something, [9] forming an opinion of the amount, value, or quality of something after thinking about it carefully, or [10] calculating the value of a function at a particular value of its independent variables. Apart from this, *forecasting* will be understood as [7] a calculation or estimate of something future, [11] a prediction of future values in a time series (where a time series is a series of measurements over time, usually at regular intervals, of a random variable, where the adjective *random* describes any process in which the results may not be certain), or [9] a statement about what will happen in the future, based on the information that is available now.

Furthermore, all the possible approaches to staff performance evaluation and forecasting will be divided into two main groups: deterministic (conditioned, predictable by results) and stochastic (probabilistic, unpredictable by results).

Achieving these goals will require a thorough study of the existing conditioned and probabilistic algorithms for processing a sequence of measured staff performance values and performing an in-depth cross-comparative analysis of these algorithms to evaluate and forecast the performance of both individual employees and professional teams of specialists, working under the same conditions, under different conditions, or both in the same area or different areas of staff activity. In this context, a comparative analysis should be conducted using a set of basic cross-comparison indicators, including comparative indicators of informativeness, accuracy, simplicity, and cost, as well as an indicator of time (an indicator of the ability to use algorithms for evaluating past performance and/or forecasting future performance).

The results of such studies and a cross-comparative analysis of deterministic and stochastic approaches to staff performance evaluation and forecasting should lead to a reasoned selection of the most effective algorithms for staff performance evaluation and forecasting, as well as the practical application conditions and features of those algorithms.

Chapter 1

Formalization of Definitions and Concepts of Staff Performance Evaluation and Forecasting

As mentioned earlier, performance measurement includes [2] the process of collecting, analyzing, and/or reporting information regarding the performance of an individual, group, organization, system, or component. Performance measurement has been defined [3] as a system that enables informed decisions to be made and actions to be taken because it quantifies the efficiency and effectiveness of past actions through the acquisition, collation, sorting, analysis and interpretation of appropriate data.

Quantitative measurement of the effectiveness of company employees (or performance evaluation and forecasting) is carried out by measuring certain performance parameters (indicators) of the corresponding measured objects [1, 4], including the use of key performance indicators [5, 6].

Each company (organization, enterprise) has its own system of indicators to measure the performance of professional staff as evaluation and forecasting objects. The structure and content of the indicators used to evaluate and forecast employee performance may vary significantly in various organizations and, as a rule, depends on the level and intensity of those enterprises' technological infrastructure, on the accepted corporate rules, on the relevant industry specialization, and on national peculiarities of business activity. Even in the same company, the indicators used for staff performance evaluation and forecasting may differ significantly in structure and content for various staff categories (positions), for various employee specializations, and for staff at different structural and regional corporate divisions. However, all these staff productivity parameters (performance standards) should be easily measurable [12].

At the same time, despite all the possible variations of staff performance evaluation and forecasting systems, for most systems, the results of employee performance evaluation and forecasting can be presented as a certain formalized parameter $\alpha_{\mathbf{rij\tau\Delta T\lambda}}$, which should characterize the quantitative value α of the corresponding measured $\mathbf{r}$-th performance indicator for some given measured $\mathbf{i}$-th employee of the corresponding $\mathbf{j}$-th enterprise at the corresponding defined time $\boldsymbol{\tau}$ for the corresponding defined measurement interval $\Delta\mathbf{T}$ in the corresponding defined unit of measurement $\boldsymbol{\lambda}$.

In this case, the performance indicator $\mathbf{r}$ $(\mathbf{r} = \overline{\mathbf{1, R}})$ should be understood as any indicator from the totality of **R** indicators used by some given **j**-th enterprise to measure the performance of its **i**-th employee. It is clear that depending on the content of the performance indicator, this parameter can be measured in different units λ: for example, dimensionless units (in unit fractions, in whole units, in tens of units, etc.), time units (in fractions of a second, in whole seconds, minutes, hours, days, weeks, months, years, etc.), monetary units (in respective currencies), distance units (in millimeters, centimeters, meters, kilometers, etc.), weight units (in grams, kilograms, tonnes, etc.), or speed units (in meters per second, kilometers per hour, etc.).

The parameter τ ($\tau = \overline{T_0, T_{max}}$) should be understood as a specific fixed measurement time (for example, the date, or where necessary, a more accurate time: hours, minutes, seconds, etc.) in the measurement time interval ΔT from T_0 to T_{max} ($\Delta T = T_{max} - T_0$), usually applying the same measurement step $\Delta t = \mathbf{constant}$. Simultaneously, the parameter τ can be assigned a time measurement procedure number in the time interval from T_0 to T_{max}, for example $\tau = \overline{1, \tau_{max}}$, where $\tau_{max} = \frac{\Delta T}{\Delta t} + 1$, provided that the measurement at time point T_0 is taken to have measurement number $\tau = 1$ and that $\Delta t = \mathbf{constant}$.

In this context, it should be noted that in certain situations, the performance indicator measurement can be linked not only to the time parameter τ, but also to other evaluation and forecasting parameters, such as distance $\mathbf{d}$ ($\mathbf{d} = \overline{D_0, D_{max}}$) parameters with the same measurement step $\Delta \mathbf{d} = \mathbf{constant}$, or financial ψ ($\psi = \overline{\Psi_0, \Psi_{max}}$) parameters with the same measurement step $\Delta\psi = \mathbf{constant}$, or other parameters. In such cases, as a rule, the assessment of the performance indicator $\alpha_{rij\tau_d \Delta T\lambda}$ (with reference to the distance parameter **d** when the same measurement step $\Delta \mathbf{d} = \mathbf{constant}$) or of the performance indicator $\alpha_{rij\tau_\psi \Delta T\lambda}$ (with reference to the financial parameter ψ when the same measurement step $\Delta\psi = \mathbf{constant}$) will occur at the appropriate certain time points τ_d or τ_ψ, applying not a constant, but a variable measurement step, $\Delta t_d \neq \mathbf{constant}$ or $\Delta t_\psi \neq \mathbf{constant}$ respectively.

Obviously, all performance indicator values **r** measured for each **i**-th employee of the corresponding **j**-th enterprise in the measurement time interval ΔT can be represented as a sequence $\mathbf{A_r}$ of values: $\alpha_{rij1\Delta T\lambda}$, $\alpha_{rij2\Delta T\lambda}$, $\ldots$, $\alpha_{rij\tau_{max}\Delta T\lambda}$. It should be immediately noted that the sequence $\mathbf{A_r}$ will henceforth be considered exclusively as a finite set, i.e., a set that does not have an infinite number of members. In most cases, for the sake of clarity, the specified measured performance indicator sequence can be reduced to a register of measurement results in the appropriate form, for example, as represented in Table 1 and/or Figure 1 below.

Table 1
Register of the absolute measured values of performance parameter **r** for the **i**-th employee of the **j**-th enterprise in the measurement time interval ΔT.

Parameter	Fixed time τ of measurement carried out with measurement step Δt				
	1	**2**	**3**	**...**	τ_{max}
r	$\alpha_{1\lambda}$	$\alpha_{2\lambda}$	$\alpha_{3\lambda}$	**...**	$\alpha_{\tau_{max}\lambda}$

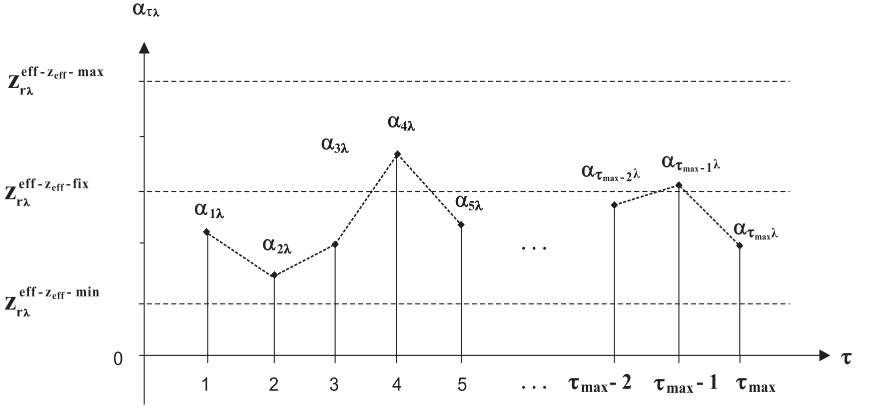

Figure 1
Absolute measured values of performance parameter **r** for the **i**-th employee of the **j**-th enterprise at measurement time interval ΔT.

The obtained initial sequence $\mathbf{A_r}$ of measured values $\alpha_{rij1\Delta T\lambda}$, $\alpha_{rij2\Delta T\lambda}$, ..., $\alpha_{rij\tau_{max}\Delta T\lambda}$ forms the basis for further calculation (measurement, assessment) of the productivity (or efficiency) of the corresponding person's activities according to the measured indicator in a certain period of time, that is, to determine the efficiency of the **i**-th employee of the **j**-th enterprise by the performance indicator **r** at measurement time interval ΔT.

According to [13], *efficiency* should be understood as the quality or property of being efficient (acting directly to produce an effect) or the degree to which this quality is exercised. According to [7], efficiency in the general sense should be understood as the state or quality of being productive at minimum waste or effort. Hence, an effective employee should be understood as a person who is capable, acting effectively and competently with the ability to accomplish or fulfill what is intended.

Using various possible data processing approaches (algorithms), the initial sequence $\mathbf{A_r}$ of measured values $\alpha_{\mathbf{rij1\Delta T\lambda}}$, $\alpha_{\mathbf{rij2\Delta T\lambda}}$, ..., $\alpha_{\mathbf{rij\tau_{max}\Delta T\lambda}}$ for performance indicator **r** of the **i**-th employee can be represented by a certain set $\mathbf{Z_{A_r}}$ of this initial sequence's characteristics $\mathbf{z_{rij\Delta T\lambda}^{A_r-1}}$, $\mathbf{z_{rij\Delta T\lambda}^{A_r-2}}$, ..., $\mathbf{z_{rij\Delta T\lambda}^{A_r-Z_{max-A_r}}}$. Each $\mathbf{z_{A_r}}$-th $(\mathbf{z_{A_r}} = \overline{\mathbf{1}, \mathbf{Z_{max-A_r}}})$ characteristic $\mathbf{z_{rij\Delta T\lambda}^{A_r-z_{A_r}}}$ of this initial sequence's characteristics set $\mathbf{Z_{A_r}}$ will be calculated according to a separate algorithm processing the measured value sequence $\mathbf{A_r}$ ($\alpha_{\mathbf{rij1\Delta T\lambda}}$, $\alpha_{\mathbf{rij2\Delta T\lambda}}$, ..., $\alpha_{\mathbf{rij\tau_{max}\Delta T\lambda}}$). For example, each characteristic may be processed by an algorithm that assesses the mean (or average value) of the set of initial sequence indices, or by algorithms that assess the maximum or minimum absolute or relative average deviation of the initial sequence indices from those indices' overall mean value, or by algorithms used to calculate the mathematical expectation, standard deviation, and similar quantities.

Based on the above, the efficient activity of the **i**-th employee of the **j**-th enterprise according to the measured performance indicator **r** in the measurement time interval $\Delta\mathbf{T}$ should be understood as the extent to which the set $\mathbf{Z_{A_r}}$ ($\mathbf{z_{rij\Delta T\lambda}^{A_r-1}}$, $\mathbf{z_{rij\Delta T\lambda}^{A_r-2}}$, ..., $\mathbf{z_{rij\Delta T\lambda}^{A_r-Z_{max-A_r}}}$) of the characteristics of the initial sequence $\mathbf{A_r}$ of the measured values $\alpha_{\mathbf{rij1\Delta T\lambda}}$, $\alpha_{\mathbf{rij2\Delta T\lambda}}$, ..., $\alpha_{\mathbf{rij\tau_{max}\Delta T\lambda}}$ achieves the reference values of the expected levels (control values, reference parameters) of the corresponding set $\mathbf{Z_{eff}}$ ($\mathbf{z_{rij\Delta T\lambda}^{eff-1}}$, $\mathbf{z_{rij\Delta T\lambda}^{eff-2}}$, ..., $\mathbf{z_{rij\Delta T\lambda}^{eff-Z_{max-eff}}}$).

Each $\mathbf{z_{eff}}$-th $(\mathbf{z_{eff}} = \overline{\mathbf{1}, \mathbf{Z_{max-eff}}})$ reference level $\mathbf{z_{rij\Delta T\lambda}^{eff-z_{eff}}}$ from the population $\mathbf{Z_{eff}}$ should be understood as a reference value defined by the corresponding **j**-th enterprise and serving as an expected performance indicator of its **i**-th employee in terms of productivity **r**.

In turn, each reference value $\mathbf{z_{rij\Delta T\lambda}^{eff-z_{eff}}}$ can be considered:

- either as the maximum possible reference value $\mathbf{z_{rij\Delta T\lambda}^{eff-z_{eff}-max}}$ of the performance indicator **r** (for example, see the absolute reference value $\mathbf{z_{r\lambda}^{eff-z_{eff}-max}}$ in Figure 1) that can be achieved by the **i**-th employee of the **j**-th enterprise in the measurement time interval $\Delta\mathbf{T}$. The maximum possible absolute reference value $\mathbf{z_{\Delta T\lambda}^{eff-z_{eff}-100\%}}$ can be illustrated, for example, by an information centre operator's 100% error-free responses to clients' incoming requests registered during the fixed time interval $\Delta\mathbf{T}$;

- or as the minimum possible value $\mathbf{z_{rij\Delta T\lambda}^{eff-z_{eff}-min}}$ of the performance indicator **r** (see, for example, the absolute reference value $\mathbf{z}_{r\lambda}^{eff-z_{eff}-min}$ in Figure 1) that can be achieved by the **i**-th employee of the **j**-th enterprise in the measurement time interval $\Delta\mathbf{T}$. The minimum possible indicator $\mathbf{z}_{\Delta T\lambda}^{eff-z_{eff}-0\%}$ can be illustrated, for example, by an information centre operator's 0% erroneous responses to clients' incoming requests registered during the fixed time interval $\Delta\mathbf{T}$;

- or as a certain fixed (expected) value $\mathbf{z}_{rij\Delta T\lambda}^{eff-z_{eff}-fix}$ of the performance indicator **r** (for example, the absolute reference value $\alpha_{r\lambda}^{eff-z_{eff}-fix}$ in Figure 1), which can be achieved by the **i**-th employee of the **j**-th enterprise in the measurement time interval $\Delta\mathbf{T}$. The feasible fixed indicator $\alpha_{\Delta T\lambda}^{eff-z_{eff}-\eta}$ can be illustrated, for example, by a planned volume η of jewelry sales made, in quantitative or monetary terms, over a time interval $\Delta\mathbf{T}$ by a jewelry store sales consultant.

The calculation of the characteristic set component values $\mathbf{Z}_{A_r}$ ($\mathbf{z}_{rij\Delta T\lambda}^{A_r-1}$, $\mathbf{z}_{rij\Delta T\lambda}^{A_r-2}$, ..., $\mathbf{z}_{rij\Delta T\lambda}^{A_r-Z_{max-A_r}}$) of the initial measured value sequence $\mathbf{A_r}$ ($\alpha_{rij1\Delta T\lambda}$, $\alpha_{rij2\Delta T\lambda}$, ..., $\alpha_{rij\tau_{max}\Delta T\lambda}$) has the goal of further determining the degree to which these characteristics' calculated values can reach the corresponding expected levels (control parameters, reference values) $\mathbf{z}_{rij\Delta T\lambda}^{eff-1}$, $\mathbf{z}_{rij\Delta T\lambda}^{eff-2}$, ..., $\mathbf{z}_{rij\Delta T\lambda}^{eff-Z_{max-eff}}$ of the set of reference values $\mathbf{Z}_{eff}$. This calculation can potentially be carried out using two main approaches: the deterministic approach and its opposite, the stochastic approach.

The deterministic approach should be understood as one that gives an outcome [11] that can be predicted exactly (or a method with a perfectly predictable outcome), one that gives a predicted result [13] because every state of affairs is the inevitable consequence of antecedent states of affairs, or one that gives a description [8] of a variable or a system's output that can be predicted with certainty.

The stochastic approach should be understood as a means of processing [11] a finite collection of related random variables, often ordered in time or space, an approach [8] that describes a variable or a system output that is subject to change or probability, or an approach [13] of, relating to, or characterized by conjecture (hypothesis) or statistics involving or containing a random variable or process.

Based on the essence of the deterministic and stochastic approaches defined above, let us consider the sequence $\mathbf{A_r}$ ($\alpha_{rij1\Delta T\lambda}$, $\alpha_{rij2\Delta T\lambda}$, ..., $\alpha_{rij\tau_{max}\Delta T\lambda}$) of measured values and the set $\mathbf{Z}_{A_r}$ of the measured values' characteristics $\mathbf{z}_{rij\Delta T\lambda}^{A_r-1}$, $\mathbf{z}_{rij\Delta T\lambda}^{A_r-2}$, ..., $\mathbf{z}_{rij\Delta T\lambda}^{A_r-Z_{A_r}}$ concurrently in two dimensions:

a) as a conditionally determined (conditioned, predictable, fixed) sequence of clearly defined (estimated) values and of the characteristics of these estimated values of the performance indicator **r** (excluding the impact of random external or internal factors on the measured personnel activities), with further processing of this sequence using appropriate deterministic approaches;

b) as a real stochastic (probabilistic, unpredictable) sequence of certain (estimated) values and of the characteristics of these estimated values of the performance indicator **r** (including the random influence of external or internal factors on the measured personnel activities), with further processing of this sequence using appropriate stochastic approaches.

Subsequent chapters of this book will present a more detailed analysis of the calculation of certain components $\mathbf{z}^{A_r-1}_{rij\Delta T\lambda}$, $\mathbf{z}^{A_r-2}_{rij\Delta T\lambda}$, …, $\mathbf{z}^{A_r-z_{max-A_r}}_{rij\Delta T\lambda}$ of the set of performance characteristics $\mathbf{Z}_{A_r}$ for the **i**-th employee of the **j**-th enterprise based on the estimated performance indicator **r** in the measurement time interval $\Delta\mathbf{T}$. These calculations will be carried out through deterministic and stochastic processing of the corresponding measured values $\alpha_{rij1\Delta T\lambda}$, $\alpha_{rij2\Delta T\lambda}$, …, $\alpha_{rij\tau_{max}\Delta T\lambda}$ of the initial sequence $\mathbf{A_r}$, with a detailed cross-comparative analysis of the deterministic and stochastic approaches used to assess staff performance characteristics by involving the main cross-comparison indicators.

Moreover, the initial sequence $\mathbf{A_r}$ of corresponding measured values $\alpha_{rij1\Delta T\lambda}$, $\alpha_{rij2\Delta T\lambda}$, …, $\alpha_{rij\tau_{max}\Delta T\lambda}$ should be found empirically, i.e., grounded [9] in practical results (based on experiments or experience rather than ideas or theories) produced by the relevant employee, not on abstract ideas or theoretical assumptions. Therefore, the more detailed analysis intended to assess the respective staff productivity degree will be carried out using descriptive statistics, which is the part of the subject of statistics concerned with describing the basic statistical features of a set of observations [10]. In other words, the basic of analysis will be to process the obtained empirical data using simple numerical descriptions, such as defining mean indicators (measures of central tendency), variation and deviation ranges (measures of spread), and forms or trends of changes (probability measures) in the measured values of the sequence $\mathbf{A_r}$, which together with the corresponding diagrams and tables, will be used to provide a clearer representation of the obtained empirical data.

Chapter 2

Analysis of the Deterministic Approach to Staff Performance Evaluation and Forecasting

Let us consider in more detail the use of a deterministic approach for staff performance evaluation and forecasting using the results of the analysis (processing) of the measured values of the sequence $\mathbf{A_r}$ ($\alpha_{rij1\Delta T\lambda}$, $\alpha_{rij2\Delta T\lambda}$, …, $\alpha_{rij\tau_{max}\Delta T\lambda}$) of the performance indicator **r**, concerning the activities of the **i**-th employee of the **j**-th enterprise at corresponding time τ in the relevant defined measurement interval $\Delta\mathbf{T}$ with the corresponding certain measurement unit λ.

Considering the deterministic approaches that are applicable to data processing, the initial sequence $\mathbf{A_r}$ of measured values can be represented by a certain set (sequence) $\mathbf{Z}_{A_r/det}$ of deterministic characteristics $\mathbf{Z}_{rij\Delta T\lambda}^{A_r/det-1}$, $\mathbf{Z}_{rij\Delta T\lambda}^{A_r/det-2}$, …, $\mathbf{Z}_{rij\Delta T\lambda}^{A_r/det-Z_{A_r/det}}$, which should be considered a subset of the previously considered set $\mathbf{Z}_{A_r}$ of deterministic and stochastic characteristics $\mathbf{Z}_{rij\Delta T\lambda}^{A_r-1}$, $\mathbf{Z}_{rij\Delta T\lambda}^{A_r-2}$, …, $\mathbf{Z}_{rij\Delta T\lambda}^{A_r-Z_{A_r}}$, that is, $\mathbf{Z}_{A_r/det} \subset \mathbf{Z}_{A_r}$.

This set $\mathbf{Z}_{A_r/det}$ can include a wide variety of deterministic characteristics that may be used to evaluate and forecast the degree of staff performance. Let us consider some of these deterministic characteristics, dividing them into three main subgroups: the mean value subgroup (**MV-D**), the variation and deviation ranges subgroup (**VDR-D**), and the subgroup of the forms and trends of changes (**FTC-D**).

1. The following deterministic characteristics will be assigned to the **MV-D** mean indicator (measures of central tendency) subgroup of the values of the measured sequence $\mathbf{A_r}$:

1.1. The arithmetic mean ("am") [10, 11, 14, 15], which should be considered as the sum of all fixed measured values ($\alpha_{rij1\Delta T\lambda}$, $\alpha_{rij2\Delta T\lambda}$, …, $\alpha_{rij\tau_{max}\Delta T\lambda}$) of sequence $\mathbf{A_r}$, divided by the total number of these measured values τ_{max}:

$$\mathbf{Z}_{rij\Delta T\lambda}^{A_r/det1-1(am)} = \frac{1}{\tau_{max}} \sum_{\tau=1}^{\tau_{max}} \alpha_{rij\tau\Delta T\lambda}. \quad (1)$$

1.2. The weighted mean ("wm") [10], which should be considered as the sum of all weighted (i.e., multiplied by the corresponding weight coefficient $\omega_{rij1\Delta T}$, $\omega_{rij2\Delta T}$, …, $\omega_{rij\tau_{max}\Delta T}$) fixed measured values ($\alpha_{rij1\Delta T\lambda}$, $\alpha_{rij2\Delta T\lambda}$, …, $\alpha_{rij\tau_{max}\Delta T\lambda}$) of sequence $\mathbf{A_r}$, divided by the sum of all these weight coefficients:

$$\mathbf{Z}_{rij\Delta T\lambda}^{A_r/det\,1-2(wm)} = \frac{\omega_{rij1\Delta T} * \alpha_{rij1\Delta T\lambda} + \omega_{rij2\Delta T} * \alpha_{rij2\Delta T\lambda} + \ldots + \omega_{rij\tau_{max}\Delta T} * \alpha_{rij\tau_{max}\Delta T\lambda}}{\omega_{rij1\Delta T} + \omega_{rij2\Delta T} + \ldots + \omega_{rij\tau_{max}\Delta T}}. \quad (2)$$

1.3. The geometric mean ("gm") [10], which should be considered as the root of the degree of the total number τ_{max} of measured values, taken of the product of all fixed measured values ($\alpha_{rij1\Delta T\lambda}$, $\alpha_{rij2\Delta T\lambda}$, …, $\alpha_{rij\tau_{max}\Delta T\lambda}$) of sequence A_r:

$$Z_{rij\Delta T\lambda}^{A_r/det\,1-3(gm)} = \sqrt[\tau_{max}]{\alpha_{rij1\Delta T\lambda} * \alpha_{rij2\Delta T\lambda} * \ldots * \alpha_{rij\tau_{max}\Delta T\lambda}}. \quad (3)$$

1.4. The harmonic mean ("hm") [10], which should be considered as the total number τ_{max} of measured values divided by the sum of the inverses of all fixed measured values ($\alpha_{rij1\Delta T\lambda}$, $\alpha_{rij2\Delta T\lambda}$, …, $\alpha_{rij\tau_{max}\Delta T\lambda}$) of sequence A_r:

$$Z_{rij\Delta T\lambda}^{A_r/det\,1-4(hm)} = \frac{\tau_{max}}{1/\alpha_{rij1\Delta T\lambda} + 1/\alpha_{rij2\Delta T\lambda} + \ldots + 1/\alpha_{rij\tau_{max}\Delta T\lambda}}. \quad (4)$$

1.5. The median ("med") [10], which should be considered as one of (or as the arithmetic mean of two) measured values of the finite sequence A_r, having an odd (or even) number of measured values, which occupies (or occupy) the medial by number (or numbers) position (or positions) in an ordered (ascending from the minimum to the maximum or, conversely, descending from the maximum to the minimum) sequence of measured values ($\alpha_{rij1\Delta T\lambda}$, $\alpha_{rij2\Delta T\lambda}$, …, $\alpha_{rij\tau_{max}\Delta T\lambda}$):

- for the case of a finite ordered sequence $A_r^{med-odd}$ with an odd number of measured values:

$$Z_{rij\Delta T\lambda}^{A_r/det\,1-5(med-odd)} = \alpha_{rij(\tau_{max}^{med}/2+0.5)\Delta T\lambda}, \quad (5)$$

where:

$(\tau_{max}^{med}/2+0.5)$ – is the average number for a member of an ordered sequence $A_r^{med-odd}$ ($\alpha_{rij1\Delta T\lambda}$, $\alpha_{rij2\Delta T\lambda}$, …, $\alpha_{rij\tau_{max}\Delta T\lambda}$) of measured values with an odd number of members;

- for a case of a finite ordered sequence $A_r^{med-even}$ with an even number of measured values:

$$Z_{rij\Delta T\lambda}^{A_r/det\,1-5(med-even)} = \frac{\alpha_{rij(\tau_{max}^{med}/2)\Delta T\lambda} + \alpha_{rij(\tau_{max}^{med}/2+1)\Delta T\lambda}}{2}, \quad (6)$$

where:

$(\tau_{max}^{med}/2)$ and $(\tau_{max}^{med}/2+1)$ are the average numbers for members of an ordered sequence $A_r^{med-even}$ of measured values ($\alpha_{rij1\Delta T\lambda}$, $\alpha_{rij2\Delta T\lambda}$, …, $\alpha_{rij\tau_{max}\Delta T\lambda}$) with an even number of members.

1.6. The mode ("mod") [10], which should be considered as the measured value (or values) of a finite sequence A_r that occurs (occur) most often among all the measured values in sequence ($\alpha_{rij1\Delta T\lambda}$, $\alpha_{rij2\Delta T\lambda}$, ..., $\alpha_{rij\tau_{max}\Delta T\lambda}$):

- for a single-modal finite sequence A_r^{mod-1}:

$$Z_{rij\Delta T\lambda}^{A_r/det\,1-6(mod-1)} = \alpha_{rij(\tau_{k-1}^{mod})\Delta T\lambda}, \tag{7}$$

where:

$\alpha_{rij(\tau_{k-1}^{mod})\Delta T\lambda}$ is the single measured value that occurs most often (namely, k times) among all measured values ($\alpha_{rij1\Delta T\lambda}$, $\alpha_{rij2\Delta T\lambda}$, ..., $\alpha_{rij\tau_{max}\Delta T\lambda}$);

- for a multi-modal (n-modal) finite sequence A_r^{mod-n}:

$$Z_{rij\Delta T\lambda}^{A_r/det\,1-6(mod-n)} = \alpha_{rij(\tau_{k-1}^{mod})\Delta T\lambda}, \ldots, \alpha_{rij(\tau_{k-n}^{mod})\Delta T\lambda}, \tag{8}$$

where:

$\alpha_{rij(\tau_{k-1}^{mod})\Delta T\lambda}, \ldots, \alpha_{rij(\tau_{k-n}^{mod})\Delta T\lambda}$ are the n measured values that occur most often (k times) among all the measured values ($\alpha_{rij1\Delta T\lambda}$, $\alpha_{rij2\Delta T\lambda}$, ..., $\alpha_{rij\tau_{max}\Delta T\lambda}$), at that $n \leq \tau_{max}$.

Special attention should be paid to the fact that most cases, the above indicators (1, 2, 3, 4, 5, 6, 7, and 8) of the **MV-D** subgroup, when applied in practice, are simply identified (considered) not as mean indicators, but as absolute indicators or simply indicators of the relevant personnel productivity (for past periods) by the corresponding parameter.

2. The following deterministic characteristics will be assigned to the variation and deviation ranges subgroup **VDR-D** (measures of spread) of the values of the measured sequence A_r:

2.1. The mean absolute deviation ("mad") [10], which should be considered as the sum of the absolute values of the differences of each fixed measured value ($\alpha_{rij1\Delta T\lambda}$, $\alpha_{rij2\Delta T\lambda}$, ..., $\alpha_{rij\tau_{max}\Delta T\lambda}$) of the sequence A_r and the mean arithmetic value $Z_{rij\Delta T\lambda}^{A_r/det\,1-1(am)}$ (1), divided by the total number τ_{max} of these measured values:

$$Z_{rij\Delta T\lambda}^{A_r/det\,2-1(mad)} = \frac{1}{\tau_{max}} \sum_{\tau=1}^{\tau_{max}} \left| \alpha_{rij\tau\Delta T\lambda} - Z_{rij\Delta T\lambda}^{A_r/det\,1-1(am)} \right|. \tag{9}$$

2.2. The mean relative deviation ("mrd"), which should be considered as the fraction obtained by dividing the mean absolute deviation $Z_{rij\Delta T\lambda}^{A_r/det\,2-1(mad)}$ (9) by the arithmetic mean $Z_{rij\Delta T\lambda}^{A_r/det\,1-1(am)}$ (1) and multiplied by 100%:

$$Z_{rij\Delta T\lambda}^{A_r/det\,2-2(mrd)} = \frac{Z_{rij\Delta T\lambda}^{A_r/det\,2-1(mad)}}{Z_{rij\Delta T\lambda}^{A_r/det\,1-1(am)}} * 100\%. \tag{10}$$

2.3. The deviation range ("ran") [10], which should be considered as the difference between the maximum (largest) and minimum (smallest) measured values among all the measured values $(\alpha_{rij1\Delta T\lambda}, \alpha_{rij2\Delta T\lambda}, \ldots, \alpha_{rij\tau_{max}\Delta T\lambda})$ of the finite sequence $\mathbf{A_r}$:

$$Z^{A_r/det\,2-3(ran)}_{rij\Delta T\lambda} = \alpha^{max}_{rij\Delta T\lambda} - \alpha^{min}_{rij\Delta T\lambda}, \quad (11)$$

where:

$\alpha^{max}_{rij\Delta T\lambda}$ and $\alpha^{min}_{rij\Delta T\lambda}$ are respectively, the maximum and minimum measured values among all the measured values $(\alpha_{rij1\Delta T\lambda}, \alpha_{rij2\Delta T\lambda}, \ldots, \alpha_{rij\tau_{max}\Delta T\lambda})$ of the finite sequence $\mathbf{A_r}$.

It is important to emphasize that the above indicators (9, 10, 11) of the **VDR-D** subgroup in most practical cases are simply identified (considered), not as variance and deviation indicators, but as indicators of the relevant staff member's performance stability (for past periods) by the corresponding parameter (e.g., the smaller the deviation factor, the greater is the staff member's performance stability as measured by the corresponding productivity parameter).

3. The following deterministic parametric and non-parametric characteristics will be assigned to the forms or trends of changes subgroup **FTC-D** (probability measures) of the values of the measured sequence $\mathbf{A_r}$:

3.1. Parametric characteristics: forecasted (expected, future, prognosticated) values $(\alpha_{rij\tau_{max+1}\Delta T\lambda}, \alpha_{rij\tau_{max+2}\Delta T\lambda}, \ldots)$ of the performance indicator **r** for the activity of the **i**-th employee of the **j**-th enterprise in the corresponding future time slots $\tau_{max+1}, \tau_{max+2}, \ldots$, respectively occurring right after τ_{max} in the corresponding defined measurement unit λ, and forecast by parametric approximation of the trend line of the measured values $(\alpha_{rij1\Delta T\lambda}, \alpha_{rij2\Delta T\lambda}, \ldots, \alpha_{rij\tau_{max}\Delta T\lambda})$ of the finite sequence $\mathbf{A_r}$. This parametric approximation uses one of the five main approximating functions $F(\alpha_{rij\tau\Delta T\lambda})$: linear, polynomial, exponential, power, or logarithmic [16], provided that the selected approximating function is as close as possible to the measured values $(\alpha_{rij1\Delta T\lambda}, \alpha_{rij2\Delta T\lambda}, \ldots, \alpha_{rij\tau_{max}\Delta T\lambda})$, e.g., as determined by the method of least squares, when $\sum_{\tau=1}^{\tau_{max}} [F(\alpha_{rij\tau\Delta T\lambda}) - \alpha_{rij\tau\Delta T\lambda}]^2 \rightarrow \mathbf{min}$. The least squares criterion involves minimizing the sum of the squares of the differences between the indicators of the corresponding approximating function and the measured values in the corresponding defined time intervals of measurement τ [16, 17].

3.1.1. The linear approximation (forecasting, prediction) function $F_{lin}(\alpha_{rij\tau\Delta T\lambda})$ of the set's trend line for all already measured values $(\alpha_{rij1\Delta T\lambda}, \alpha_{rij2\Delta T\lambda}, \ldots, \alpha_{rij\tau_{max}\Delta T\lambda})$ of the finite sequence $\mathbf{A_r}$:

$$F_{lin}(\alpha_{rij\tau\Delta T\lambda}) = a_{lin1} * \tau + a_{lin0}, \quad (12)$$

where:

a_{lin0}, a_{lin1} are constant linear trend coefficients, known respectively as the intercept coefficient and the slope coefficient [16, 17], and calculated as follows [16, 17, 18]:

$$a_{lin0} = F_{linmean}(\alpha_{rij\Delta T\lambda}) - a_{lin1} * \tau_{mean};$$
$$a_{lin1} = \frac{\sum_{\tau=1}^{\tau_{max}}(\tau - \tau_{mean}) * [F_{lin}(\alpha_{rij\tau\Delta T\lambda}) - F_{linmean}(\alpha_{rij\Delta T\lambda})]}{\sum_{\tau=1}^{\tau_{max}}(\tau - \tau_{mean})^2} =$$
$$= \frac{\tau_{max} * \sum_{\tau=1}^{\tau_{max}} \tau * F_{lin}(\alpha_{rij\tau\Delta T\lambda}) - [\sum_{\tau=1}^{\tau_{max}} \tau] * [\sum_{\tau=1}^{\tau_{max}} F_{lin}(\alpha_{rij\tau\Delta T\lambda})]}{\sum_{\tau=1}^{\tau_{max}} \tau^2 - [\sum_{\tau=1}^{\tau_{max}} F_{lin}(\alpha_{rij\tau\Delta T\lambda})]^2};$$
$$F_{linmean}(\alpha_{rij\Delta T\lambda}) = \frac{1}{\tau_{max}} * \sum_{\tau=1}^{\tau_{max}} F_{lin}(\alpha_{rij\tau\Delta T\lambda});$$
$$\tau_{mean} = \frac{1}{\tau_{max}} * \sum_{\tau=1}^{\tau_{max}} \tau.$$

3.1.2. The polynomial approximation (forecasting, prediction) function $F_{pol}(\alpha_{rij\tau\Delta T\lambda})$ of the set's trend line for all already measured values $(\alpha_{rij1\Delta T\lambda}, \alpha_{rij2\Delta T\lambda}, \ldots, \alpha_{rij\tau_{max}\Delta T\lambda})$ of the finite sequence $\mathbf{A_r}$:

$$F_{pol}(\alpha_{rij\tau\Delta T\lambda}) = a_{poln} * \tau^n + \ldots + a_{pol1} * \tau + a_{pol0}, \tag{13}$$

where:

a_{pol0}, a_{pol1}, …, a_{poln} are the constant polynomial trend coefficients calculated using, for example, Cramer's models or the Gauss-Seidel method for solving the following system of linear equations [18, 19]:

$$\begin{cases} a_{pol0} * \tau_{max} + a_{pol1} * \sum_{\tau=1}^{\tau_{max}} \tau + \ldots + a_{poln} * \sum_{\tau=1}^{\tau_{max}} \tau^n = \sum_{\tau=1}^{\tau_{max}} [F_{pol}(\alpha_{rij\tau\Delta T\lambda})] \\ a_{pol0} * \sum_{\tau=1}^{\tau_{max}} \tau + a_{pol1} * \sum_{\tau=1}^{\tau_{max}} \tau^2 + \ldots + a_{poln} * \sum_{\tau=1}^{\tau_{max}} \tau^{n+1} = \sum_{\tau=1}^{\tau_{max}} [F_{pol}(\alpha_{rij\tau\Delta T\lambda}) * \tau] \\ \ldots + \ldots + \ldots + \ldots = \ldots \\ a_{pol0} * \sum_{\tau=1}^{\tau_{max}} \tau^n + a_{pol1} * \sum_{\tau=1}^{\tau_{max}} \tau^{n+1} + \ldots + a_{poln} * \sum_{\tau=1}^{\tau_{max}} \tau^{2n} = \sum_{\tau=1}^{\tau_{max}} [F_{pol}(\alpha_{rij\tau\Delta T\lambda}) * \tau^n] \end{cases}.$$

3.1.3. The exponential $F_e(\alpha_{rij\tau\Delta T\lambda})$ approximation (forecasting, prediction) function for the trend line of the set of all measured values $(\alpha_{rij1\Delta T\lambda}, \alpha_{rij2\Delta T\lambda}, \ldots, \alpha_{rij\tau_{max}\Delta T\lambda})$ of the finite sequence $\mathbf{A_r}$ [16, 18]:

$$F_{e1}(\alpha_{rij\tau\Delta T\lambda}) = a_{e0-1} * e^{a_{e1-1} * \tau} \quad \text{or} \tag{14}$$
$$F_{e2}(\alpha_{rij\tau\Delta T\lambda}) = a_{e0-2} * a_{e1-2}^{\ \tau}. \tag{15}$$

After appropriate mathematical transformations:

$$F_{e1}^*(\alpha_{rij\tau\Delta T\lambda}) = a_{e0-1}^* + a_{e1-1} * \tau \quad \text{or} \tag{16}$$
$$F_{e2}^*(\alpha_{rij\tau\Delta T\lambda}) = a_{e0-2}^* + a_{e1-2}^* * \tau, \tag{17}$$

where:

$F_{e1}^*(\alpha_{rij\tau\Delta T\lambda}) = \ln[F_{e1}(\alpha_{rij\tau\Delta T\lambda})]$ is the reduced (reduced to linear) exponential approximation function (14);

$F_{e2}^*(\alpha_{rij\tau\Delta T\lambda}) = \log[F_{e2}(\alpha_{rij\tau\Delta T\lambda})]$ is the reduced (reduced to linear) exponential approximation function (15);

$\mathbf{a}^*_{e0-1} = \ln[\mathbf{a}_{e0-1}]$ is a constant coefficient of an exponential trend reduced to linear (14), known as the coefficient of the abscissa line intersection (intercept coefficient);

$\mathbf{a}^*_{e0-2} = \log[\mathbf{a}_{e0-2}]$ is a constant coefficient of an exponential trend reduced to linear (15), known as the coefficient of the abscissa line crossing (intercept coefficient);

$\mathbf{a}_{e1-1}$ is a constant coefficient of an exponential trend reduced to linear (14), known as the slope coefficient;

$\mathbf{a}^*_{e1-2} = \log(\mathbf{a}_{e1-2})$ is a constant coefficient of an exponential trend reduced to linear (15), known as the slope coefficient;

$$\mathbf{a}^*_{e0-1} = \mathbf{F}^*_{e1mean}(\alpha_{rij\Delta T\lambda}) - \mathbf{a}_{e1-1} * \tau_{mean};$$

$$\mathbf{a}^*_{e0-2} = \mathbf{F}^*_{e2mean}(\alpha_{rij\Delta T\lambda}) - \mathbf{a}_{e1-2} * \tau_{mean};$$

$$\mathbf{a}_{e1-1} = \frac{\sum_{\tau=1}^{\tau_{max}} (\tau - \tau_{mean}) * [\mathbf{F}^*_{e1}(\alpha_{rij\tau\Delta T\lambda}) - \mathbf{F}^*_{e1mean}(\alpha_{rij\Delta T\lambda})]}{\sum_{\tau=1}^{\tau_{max}} (\tau - \tau_{mean})^2};$$

$$\mathbf{a}_{e1-2} = \frac{\sum_{\tau=1}^{\tau_{max}} (\tau - \tau_{mean}) * [\mathbf{F}^*_{e2}(\alpha_{rij\tau\Delta T\lambda}) - \mathbf{F}^*_{e2mean}(\alpha_{rij\Delta T\lambda})]}{\sum_{\tau=1}^{\tau_{max}} (\tau - \tau_{mean})^2};$$

$$\mathbf{F}^*_{e1mean}(\alpha_{rij\Delta T\lambda}) = \frac{1}{\tau_{max}} * \sum_{\tau=1}^{\tau_{max}} \mathbf{F}^*_{e1}(\alpha_{rij\tau\Delta T\lambda});$$

$$\mathbf{F}^*_{e2mean}(\alpha_{rij\Delta T\lambda}) = \frac{1}{\tau_{max}} * \sum_{\tau=1}^{\tau_{max}} \mathbf{F}^*_{e2}(\alpha_{rij\tau\Delta T\lambda});$$

$$\tau_{mean} = \frac{1}{\tau_{max}} * \sum_{\tau=1}^{\tau_{max}} \tau.$$

3.1.4. The power $\mathbf{F}_{pow}(\alpha_{rij\tau\Delta T\lambda})$ approximation (forecasting, prediction) function for the trend line of the set of all measured values $(\alpha_{rij1\Delta T\lambda}, \alpha_{rij2\Delta T\lambda}, \ldots, \alpha_{rij\tau_{max}\Delta T\lambda})$ of the finite sequence $\mathbf{A_r}$ [18]:

$$\mathbf{F}_{pow}(\alpha_{rij\tau\Delta T\lambda}) = \mathbf{a}_{pow0} * \tau^{\mathbf{a}_{pow1}}, \tag{18}$$

or after appropriate mathematical transformations:

$$\mathbf{F}^*_{pow}(\alpha_{rij\tau^*\Delta T\lambda}) = \mathbf{a}^*_{pow0} + \mathbf{a}_{pow1} * \tau^*, \tag{19}$$

where:

$\mathbf{F}^*_{pow}(\alpha_{rij\tau^*\Delta T\lambda}) = \log[\mathbf{F}_{pow}(\alpha_{rij\tau\Delta T\lambda})]$ is the reduced (reduced to linear) power approximation function;

$\tau^* = \log(\tau)$ is the variable of the function for the reduced-to-linear power trend;

$\mathbf{a}^*_{pow0} = \log(\mathbf{a}_{pow0})$ is the constant coefficient of the reduced-to-linear power trend, known as the coefficient of the abscissa line crossing (intercept coefficient);

$\mathbf{a}_{pow1}$ is the constant coefficient of the reduced-to-linear power trend, known as the slope coefficient;

$$\mathbf{a}^*_{pow0} = \mathbf{F}^*_{powmean}(\alpha_{rij\Delta T\lambda}) - \mathbf{a}_{pow1} * \tau^*_{mean};$$

$$\mathbf{a}_{pow1} = \frac{\sum_{\tau^*=\log(1)}^{\tau^*_{max}} (\tau^* - \tau^*_{mean}) * [\mathbf{F}^*_{pow}(\alpha_{rij\tau^*\Delta T\lambda}) - \mathbf{F}^*_{powmean}(\alpha_{rij\Delta T\lambda})]}{\sum_{\tau^*=\log(1)}^{\tau^*_{max}} (\tau^* - \tau^*_{mean})^2};$$

$$\mathbf{F}^{*}_{\mathbf{powmean}}(\alpha_{\mathbf{rij}\Delta\mathrm{T}\lambda}) = \frac{1}{\tau^{*}_{\mathbf{max}}} * \sum_{\tau^{*}=\log(1)}^{\tau^{*}_{max}} \mathbf{F}^{*}_{\mathbf{pow}}(\alpha_{\mathbf{rij}\tau^{*}\Delta\mathrm{T}\lambda});$$

$$\tau^{*}_{\mathbf{mean}} = \frac{1}{\tau^{*}_{\mathbf{max}}} * \sum_{\tau^{*}=\log(1)}^{\tau^{*}_{max}} \tau^{*}.$$

3.1.5. The logarithmic $\mathbf{F}_{\mathbf{ln}}(\alpha_{\mathbf{rij}\tau\Delta\mathrm{T}\lambda})$ approximation (forecasting, prediction) function for the trend line of the set of all measured values $(\alpha_{\mathbf{rij}1\Delta\mathrm{T}\lambda}, \alpha_{\mathbf{rij}2\Delta\mathrm{T}\lambda}, \ldots, \alpha_{\mathbf{rij}\tau_{max}\Delta\mathrm{T}\lambda})$ of the finite sequence $\mathbf{A_r}$:

$$\mathbf{F}_{\mathbf{ln}}(\alpha_{\mathbf{rij}\tau\Delta\mathrm{T}\lambda}) = \mathbf{a}_{\mathbf{ln0}} + \mathbf{a}_{\mathbf{ln1}} * \mathbf{ln}(\tau), \quad (20)$$

or after appropriate mathematical transformations:

$$\mathbf{F}^{*}_{\mathbf{ln}}(\alpha_{\mathbf{rij}\tau^{*}\Delta\mathrm{T}\lambda}) = \mathbf{a}_{\mathbf{ln0}} + \mathbf{a}_{\mathbf{ln1}} * \tau^{*}, \quad (21)$$

where:

$\mathbf{F}^{*}_{\mathbf{ln}}(\alpha_{\mathbf{rij}\tau^{*}\Delta\mathrm{T}\lambda})$ is the reduced (reduced to linear) logarithmic approximation function;

$\tau^{*} = \mathbf{ln}(\tau)$ is the variable of the function for the reduced-to-linear logarithmic trend;

$\mathbf{a}_{\mathbf{ln0}}$ is the logarithmic trend constant coefficient, known as the coefficient of the abscissa line crossing (intercept coefficient);

$\mathbf{a}_{\mathbf{ln1}}$ is the logarithmic trend constant coefficient, known as the slope coefficient;

$$\mathbf{a}_{\mathbf{ln0}} = \mathbf{F}^{*}_{\mathbf{ln}}(\alpha_{\mathbf{rij}\tau^{*}\Delta\mathrm{T}\lambda}) - \mathbf{a}_{\mathbf{ln}} * \tau^{*}_{\mathbf{mean}};$$

$$\mathbf{a}_{\mathbf{lin1}} = \frac{\sum_{\tau^{*}=\ln(1)}^{\tau^{*}_{max}} (\tau^{*} - \tau^{*}_{\mathbf{mean}}) * [\mathbf{F}_{\mathbf{ln}}(\alpha_{\mathbf{rij}\tau^{*}\Delta\mathrm{T}\lambda}) - \mathbf{F}^{*}_{\mathbf{lnmean}}(\alpha_{\mathbf{rij}\Delta\mathrm{T}\lambda})]}{\sum_{\tau^{*}=\ln(1)}^{\tau^{*}_{max}} = (\tau^{*} - \tau^{*}_{\mathbf{mean}})^{2}};$$

$$\mathbf{F}^{*}_{\mathbf{lnmean}}(\alpha_{\mathbf{rij}\Delta\mathrm{T}\lambda}) = \frac{1}{\tau^{*}_{\mathbf{max}}} * \sum_{\tau^{*}=\ln(1)}^{\tau^{*}_{max}} \mathbf{F}^{*}_{\mathbf{ln}}(\alpha_{\mathbf{rij}\tau^{*}\Delta\mathrm{T}\lambda});$$

$$\tau^{*}_{\mathbf{mean}} = \frac{1}{\tau^{*}_{\mathbf{max}}} * \sum_{\tau^{*}=\ln(1)}^{\tau^{*}_{max}} \tau^{*}.$$

3.2. Non-parametric characteristics are the forecasted (expected, future, prognosticated) values $(\alpha_{\mathbf{rij}\tau_{max+1}\Delta\mathrm{T}\lambda})$ of the performance indicator **r** representing the activity of the **i**-th employee of the **j**-th enterprise at the corresponding future time point $\tau_{\mathbf{max+1}}$. These values will accordingly be forecasted in time after $\tau_{\mathbf{max}}$, are measurable with the corresponding unit of measurement λ, and can be assessed by non-parametric approximation by smoothing the measured values $(\alpha_{\mathbf{rij}1\Delta\mathrm{T}\lambda}, \alpha_{\mathbf{rij}2\Delta\mathrm{T}\lambda}, \ldots, \alpha_{\mathbf{rij}\tau_{max}\Delta\mathrm{T}\lambda})$ of the finite sequence $\mathbf{A_r}$, using in particular the methods of simple [17] or weighted [20, 21] moving averages and exponential smoothing [17]. This can be done provided [22] that the confidence interval $\Delta\alpha_{\mathbf{rij}\tau_{max+1}\Delta\mathrm{T}\lambda}$ is minimized, that is, when $\Delta\alpha_{\mathbf{rij}\tau_{max+1}\Delta\mathrm{T}\lambda} = \nu_{\tau,\mathbf{P}} * \mathbf{s}_{\tau,\alpha} \rightarrow \mathbf{min}$ [for the corresponding value $\nu_{\tau,\mathbf{P}}$ of the Student's distribution with τ (namely, $\tau_{\mathbf{max}} - \mathbf{1}$) degrees of freedom and for the selected confidence probability level $\mathbf{P}$ (namely, $\mathbf{P}_{\Delta\alpha_{\mathbf{rij}\tau_{max+1}\Delta\mathrm{T}\lambda}}$), as well as for the reduced value $\mathbf{s}_{\tau,\alpha}$ $(\mathbf{s}_{\tau,\alpha} = \frac{\mathbf{S}_{\tau,\alpha}}{\sqrt{\tau_{\mathbf{sample}}}})$

of the standard deviation $S_{\tau,\alpha}$ ($S_{\tau,\alpha} = \sqrt{\frac{\sum_{\tau=\tau_{max}-\tau_{sample}+1}^{\tau_{max}}(\alpha_{rij\tau\Delta T\lambda} - \alpha_{rij\tau_{max+1}\Delta T\lambda})^2}{\tau_{sample}-1}}$) for each value from the sample τ_{sample} of measured values ($\alpha_{rij(\tau_{max}-\tau_{sample}+1)\Delta T\lambda}$, $\alpha_{rij(\tau_{max}-\tau_{sample}+2)\Delta T\lambda}$, …, $\alpha_{rij\tau_{max}\Delta T\lambda}$) and the forecasted value $\alpha_{rij\tau_{max+1}\Delta T\lambda}$].

3.2.1. The method of simple moving average smoothing (forecast, approximation, prediction) of all or part (sample τ_{sample}) of all measured values ($\alpha_{rij(\tau_{max}-\tau_{sample}+1)\Delta T\lambda}$, $\alpha_{rij(\tau_{max}-\tau_{sample}+2)\Delta T\lambda}$, …, $\alpha_{rij\tau_{max}\Delta T\lambda}$) of the finite sequence $\mathbf{A_r}$, which can determine (forecast) the value $\alpha_{rij\tau_{max+1}\Delta T\lambda}$ for the future time point τ_{max+1}:

$$\hat{z}_{rij\tau_{max+1}\Delta T\lambda} = \frac{\sum_{\tau=\tau_{max}-\tau_{sample}+1}^{\tau_{max}} \alpha_{rij\tau\Delta T\lambda}}{\tau_{sample}}, \tag{22}$$

where:

$\hat{z}_{rij\tau_{max+1}\Delta T\lambda}$ is the forecasted (predicted, estimated) value $\alpha_{rij\tau_{max+1}\Delta T\lambda}$ of the performance indicator **r** at future time point τ_{max+1}, constructed using the simple moving average method;

τ_{sample} is the sample size of the most recently measured values ($\alpha_{rij1\Delta T\lambda}$, $\alpha_{rij2\Delta T\lambda}$, …, $\alpha_{rij\tau_{sample}\Delta T\lambda}$), used to forecast (obtain an estimate of) the value $\alpha_{rij\tau_{max+1}\Delta T\lambda}$, when $\tau_{sample} \le \tau_{max}$.

3.2.2. The method of weighted moving average smoothing (forecast, approximation, prediction) of all or part (sample τ_{sample}) of all weighted (i.e., multiplied by the corresponding values of weights or weight coefficients $\omega_{rij(\tau_{max}-\tau_{sample}+1)\Delta T}$, $\omega_{rij(\tau_{max}-\tau_{sample}+2)\Delta T}$, …, $\omega_{rij\tau_{max}\Delta T}$) of the fixed values ($\alpha_{rij(\tau_{max}-\tau_{sample}+1)\Delta T\lambda}$, $\alpha_{rij(\tau_{max}-\tau_{sample}+2)\Delta T\lambda}$, …, $\alpha_{rij\tau_{max}\Delta T\lambda}$) of the finite sequence $\mathbf{A_r}$, which can determine (forecast) the value $\alpha_{rij\tau_{max+1}\Delta T\lambda}$ for the future time point τ_{max+1}:

$$\hat{\hat{z}}_{rij\tau_{max+1}\Delta T\lambda} = \frac{\sum_{\tau=\tau_{max}-\tau_{sample}+1}^{\tau_{max}} \omega_{rij\tau\Delta T\lambda} * \alpha_{rij\tau\Delta T\lambda}}{\tau_{sample}}, \tag{23}$$

where:

$\hat{\hat{z}}_{rij\tau_{max+1}\Delta T\lambda}$ is the forecasted (predicted, estimated) value $\alpha_{rij\tau_{max+1}\Delta T\lambda}$ of the performance indicator **r** at future time point τ_{max+1}, constructed using the weighted moving average method.

3.2.3. The method of exponential smoothing (forecast, approximation, prediction) of the fixed values ($\alpha_{rij(\tau_{max}-\tau_{sample}+1)\Delta T\lambda}$, $\alpha_{rij(\tau_{max}-\tau_{sample}+2)\Delta T\lambda}$, …, $\alpha_{rij\tau_{max}\Delta T\lambda}$) of the finite sequence $\mathbf{A_r}$, which can determine (forecast) the value $\alpha_{rij\tau_{max+1}\Delta T\lambda}$ for the future time point τ_{max+1}:

$$\tilde{z}_{rij\tau_{max+1}\Delta T\lambda} = \theta * \alpha_{rij\tau_{max}\Delta T\lambda} + (1-\theta) * \tilde{z}_{rij\tau_{max}\Delta T\lambda}, \tag{24}$$

where:

$\tilde{z}_{rij\tau_{max+1}\Delta T\lambda}$ is the forecasted (predicted, estimated) value $\alpha_{rij\tau_{max+1}\Delta T\lambda}$ of the performance indicator **r** at future time point τ_{max+1}, constructed using exponential smoothing;

$\tilde{\mathbf{z}}_{\mathbf{rij}\tau_{\max}\Delta\mathbf{T}\lambda}$ is the forecasted (predicted, estimated) value $\alpha_{\mathbf{rij}\tau_{\max}\Delta\mathbf{T}\lambda}$ of the performance indicator **r** at the past time point $\tau_{\mathbf{max}}$, constructed using exponential smoothing;

θ is a smoothing constant $(0 \leq \theta \leq 1)$.

It is essential to emphasize that the **FTC-D** subgroup's indicators (12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, and 24) referred to above are in most practical application cases identified (considered) for the sake of simplicity, not as indicators of the forms or trends of changes, but as absolute forecasted (expected) indicators or simply forecasted (expected) indicators of the performance of the relevant staff (in future periods) for the relevant parameter.

Chapter 3

Analysis of the Stochastic Approach to Staff Performance Evaluation and Forecasting

Let us consider in further detail the application of the stochastic approach for staff performance evaluation and forecasting based on the results of analyzing (processing) sequence $\mathbf{A_r}$ of measured values ($\alpha_{rij1\Delta T\lambda}$, $\alpha_{rij2\Delta T\lambda}$, …, $\alpha_{rij\tau_{max}\Delta T\lambda}$) of performance indicator **r** of the activities of the **i**-th employee of the **j**-th enterprise at corresponding time $\boldsymbol{\tau}$ in the relevant defined measurement interval $\Delta\mathbf{T}$ with the respective certain measurement unit $\boldsymbol{\lambda}$.

By using stochastic data processing approaches, the initial sequence of measured values $\mathbf{A_r}$ can be represented by a certain set (sequence) $\mathbf{Z}_{A_r/stoch}$ of stochastic characteristics $\mathbf{z}_{rij\Delta T\lambda}^{A_r/stoch-1}$, $\mathbf{z}_{rij\Delta T\lambda}^{A_r/stoch-2}$, …, $\mathbf{z}_{rij\Delta T\lambda}^{A_r/stoch-Z_{A_r/stoch}}$, which should be considered as a subset of the earlier investigated set $\mathbf{Z}_{A_r}$ of deterministic and stochastic characteristics $\mathbf{z}_{rij\Delta T\lambda}^{A_r-1}$, $\mathbf{z}_{rij\Delta T\lambda}^{A_r-2}$, …, $\mathbf{z}_{rij\Delta T\lambda}^{A_r-Z_{A_r}}$, that is, $\mathbf{Z}_{A_r/stoch} \subset \mathbf{Z}_{A_r}$.

The specified set $\mathbf{Z}_{A_r/stoch}$ may include a certain variety of stochastic characteristics that can be used to evaluate and forecast staff performance. Let us consider some of these stochastic characteristics and divide them into three main subgroups: the subgroup of mean values (the **MV-S** subgroup of absolute indicators), the subgroup of variation and deviation ranges (the **VDR-S** subgroup of stability indicators), and the subgroup of the forms or trends of changes (the **FTC-S** subgroup of expected performance values).

1. The following stochastic characteristics will be assigned to the **MV-S** subgroup of the mean indicators (measures of central tendency) of the measured sequence values $\mathbf{A_r}$:

1.1. The mathematical expectation (“me”) [23, 24], which should be considered as the sum of the products of each of the possible values ($\alpha_{1-rij\Delta T\lambda}$, $\alpha_{2-rij\Delta T\lambda}$, …, $\alpha_{\alpha^*_{max}-rij\Delta T\lambda}$) of the set $\mathbf{A_r^*}$ for a corresponding measured performance indicator **r** from among all the fixed measured values ($\alpha_{rij1\Delta T\lambda}$, $\alpha_{rij2\Delta T\lambda}$, …, $\alpha_{rij\tau_{max}\Delta T\lambda}$) of the sequence $\mathbf{A_r}$, each possible value being multiplied by the probability of occurrence of that value:

$$\mathbf{z}_{rij\Delta T\lambda}^{A_r/stoch1-1(me)} = \sum_{\alpha^*=1}^{\alpha^*_{max}} \alpha_{\alpha^*-rij\Delta T\lambda} * \mathbf{p}_{\alpha^*-rij\Delta T\lambda}, \quad (25)$$

where:

α^* is the α^*-th number of distinct values of the corresponding performance indicator among all its possible measured values, $\alpha^* = \overline{1, \alpha^*_{max}}$, $\alpha^*_{max} \le \boldsymbol{\tau}_{max}$;

$\alpha_{\alpha^*-rij\Delta T\lambda}$ is the α^* - th distinct value of the corresponding performance indicator among all its possible measured values, $\alpha_{1-rij\Delta T\lambda} < < \alpha_{2-rij\Delta T\lambda} < \ldots < \alpha_{\alpha^*_{max}-rij\Delta T\lambda}$;

$\mathbf{A}^*_{\mathbf{r}}$ is the sequence (set) of all possible values of the corresponding performance indicator, $\mathbf{A}^*_{\mathbf{r}} = \{\alpha_{1-rij\Delta T\lambda}, \alpha_{2-rij\Delta T\lambda}, \ldots, \alpha_{\alpha^*_{max}rij\Delta T\lambda}\}$, and sequence $\mathbf{A_r}$ is a subset of the population $\mathbf{A}^*_{\mathbf{r}}$, $\mathbf{A}^*_{\mathbf{r}} \supset \mathbf{A_r}$;

$p_{\alpha^*-rij\Delta T\lambda}$ is the probability that the relevant performance indicator will acquire a possible value $\alpha_{\alpha^*-rij\Delta T\lambda}$, $p_{\alpha^*-rij\Delta T\lambda} = \frac{k_{\alpha^*-rij\Delta T\lambda}}{\tau_{max}}$;

$k_{\alpha^*-rij\Delta T\lambda}$ is the number of measured values from the sequence $\mathbf{A_r}$ that are equal to the value $\alpha_{\alpha^*-rij\Delta T\lambda}$ in the set $\mathbf{A}^*_{\mathbf{r}}$, $\sum_{\alpha^*=1}^{\alpha^*_{max}} k_{\alpha^*-rij\Delta T\lambda} = \tau_{max}$.

It is emphasized here that the mathematical expectation as defined above (25) for the subgroup **MV-S** in most practical cases is simply considered, not as a mean indicator, but as an absolute indicator or simply an indicator of relevant past and future staff productivity by the corresponding parameter.

Note that the possible values of the measured performance indicator **r** in sequence $\mathbf{A}^*_{\mathbf{r}}$, as well as the sequence $\mathbf{A_r}$ of the measured values of this indicator, will be considered exclusively as a finite set, i.e., a set that does not have an infinite number of members. In most cases, the set of measured values $\mathbf{A_r}$, as well as the corresponding numbers of measured values $k_{\alpha^*-rij\Delta T\lambda}$, which are equal to the values of $\mathbf{A}^*_{\mathbf{r}}$, can be summarized in an appropriate form for measurement results, for example, in the form shown in Table 2 and/or in Figure (histogram) 2.

Table 2
Numbers of measured performance indicators **r**
for the **i**-th employee of the **j**-th enterprise
according to possible values of this performance indicator.

Parameter	Parameter value				
Value of indicator **r**	$\alpha_{1-rij\Delta T\lambda}$	$\alpha_{2-rij\Delta T\lambda}$	$\alpha_{3-rij\Delta T\lambda}$	...	$\alpha_{\alpha^*_{max}-rij\Delta T\lambda}$
Number of indicators **r**	$k_{1-rij\Delta T\lambda}$	$k_{2-rij\Delta T\lambda}$	$k_{3-rij\Delta T\lambda}$	...	$k_{\alpha^*_{max}-rij\Delta T\lambda}$

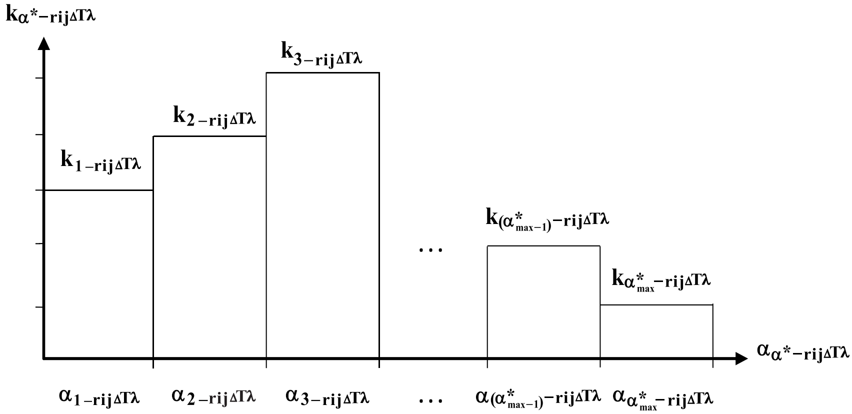

Figure (histogram) 2
Number of measured performance indicators **r**
for the **i**-th employee of the **j**-th enterprise
according to possible values of this performance indicator.

In this book, a *histogram* [10, 11, 25, 26] is defined as a diagram (graph) consisting of rectangles (columns), where the height (value on the vertical axis) of each column is equal to the number of occurrences of possible values of the measured indicator **r** (i.e., the value $k_{\alpha^*-rij\Delta T\lambda}$), and where the width (range along the horizontal axis) is equal to the range of possible values of the measured indicator **r** (i.e., the value $\alpha_{\alpha^*-rij\Delta T\lambda}$).

2. The following stochastic characteristics will be assigned to the subgroup of variation and deviation ranges **VDR-S** (measures of spread) of the values of the measured sequence $\mathbf{A_r}$:

2.1. The standard deviation ("sd") [23], which should be considered as the square root of the sum of squared differences of all possible values $(\alpha_{1-rij\Delta T\lambda}, \alpha_{2-rij\Delta T\lambda}, \ldots, \alpha_{\alpha^*_{max}-rij\Delta T\lambda})$ of the corresponding measured performance indicator **r** in sequence $\mathbf{A^*_r}$ among all the measured fixed values $(\alpha_{rij1\Delta T\lambda}, \alpha_{rij2\Delta T\lambda}, \ldots, \alpha_{rij\tau_{max}\Delta T\lambda})$ belonging to sequence $\mathbf{A_r}$, minus the corresponding mathematical expectation $\mathbf{z}_{rij\Delta T\lambda}^{A_r/stoch1-1(me)}$ (25). Each squared difference term is multiplied by the probability of occurrence of the possible value:

$$\mathbf{z}_{rij\Delta T\lambda}^{A_r/stoch2-1(sd)} = \sqrt{\sum_{\alpha^*=1}^{\alpha^*_{max}} \left[\left(\alpha_{\alpha^*-rij\Delta T\lambda} - \mathbf{z}_{rij\Delta T\lambda}^{A_r/stoch1-1(me)}\right)^2 * p_{\alpha^*-rij\Delta T\lambda} \right]}. \quad (26)$$

2.2. The coefficient of variation ("cv") [23], which should be considered as the ratio of the standard deviation $\mathbf{z}_{rij\Delta T\lambda}^{A_r/stoch2-1(sd)}$ (26), divided by the arithmetic mean $z_{rij\Delta T\lambda}^{A_r/det1-1(am)}$ (1) or by the mathematical expectation $z_{rij\Delta T\lambda}^{A_r/stoch1-1(me)}$ (25) and sequentially multiplied by 100%:

$$z_{rij\Delta T\lambda}^{A_r/stoch2-2(cv)} = \frac{z_{rij\Delta T\lambda}^{A_r/stoch2-1(sd)}}{z_{rij\Delta T\lambda}^{A_r/det1-1(am)}} * 100\% = \frac{z_{rij\Delta T\lambda}^{A_r/stoch2-1(sd)}}{z_{rij\Delta T\lambda}^{A_r/stoch1-1(me)}} * 100\%. \quad (27)$$

It is once again emphasized that in most practical cases, the above indicators (26) and (27) of the **VDR-S** subgroup are simply considered, not as variance and deviation indicators, but as measures of the relevant staff member's past and future performance stability by the corresponding parameter (e.g., the smaller the deviation factor, the higher will be the staff member's performance stability, as measured by the corresponding productivity parameter).

3. The following stochastic characteristics will be assigned to the forms or trends of changes subgroup **FTC-S** (probability measures) of the measured value sequence $\mathbf{A_r}$:

3.1. Karl Pearson's coefficient of skewness ("cs") [23], which should be considered as the ratio of the third-order central moment $\mu_{3\alpha-rij\Delta T\lambda}$ of all possible values ($\alpha_{1-rij\Delta T\lambda}, \alpha_{2-rij\Delta T\lambda}, \ldots, \alpha_{\alpha^*_{max}-rij\Delta T\lambda}$) of the corresponding measured performance indicator sequence $\mathbf{A_r^*}$, divided by the cube of the standard deviation $z_{rij\Delta T\lambda}^{A_r/stoch2-1(sd)}$ (26):

$$z_{rij\Delta T\lambda}^{A_r/stoch3-1(cs)} = \frac{\mu_{3\alpha-rij\Delta T\lambda}}{[z_{rij\Delta T\lambda}^{A_r/stoch2-1(sd)}]^3}, \quad (28)$$

where:

$$\mu_{3\alpha-rij\Delta T\lambda} = \sum_{\alpha^*=1}^{\alpha^*_{max}} [(\alpha_{\alpha^*-rij\Delta T\lambda} - z_{rij\Delta T\lambda}^{A_r/stoch1-1(me)})^3 * p_{\alpha^*-rij\Delta T\lambda}].$$

Note that Karl Pearson's coefficient of skewness (28) in the **FTC-S** subgroup is in most practical cases simply considered, not as an indicator of the forms or trends of changes, but rather as a measure of the past and future prospects for a staff member's performance to improve or deteriorate according to the corresponding parameter (e.g., a negative value of the skewness coefficient indicates a prospect of an increase in the personnel activity absolute performance indicator by the corresponding productivity parameter, and vice versa).

3.2. Karl Pearson's coefficient of kurtosis ("ck") [23], which should be considered as the ratio of the fourth-order central moment $\mu_{4\alpha-rij\Delta T\lambda}$ of all possible values ($\alpha_{1-rij\Delta T\lambda}, \alpha_{2-rij\Delta T\lambda}, \ldots, \alpha_{\alpha^*_{max}-rij\Delta T\lambda}$) of the corresponding measured performance indicator sequence $\mathbf{A_r^*}$, divided by the fourth power of the standard deviation $z_{rij\Delta T\lambda}^{A_r/stoch2-1(sd)}$ (26) and reduced by 3 units:

$$z_{rij\Delta T\lambda}^{A_r/stoch3-2(ck)} = \frac{\mu_{4\alpha-rij\Delta T\lambda}}{[z_{rij\Delta T\lambda}^{A_r/stoch2-1(sd)}]^4} - 3, \quad (29)$$

where:

$$\mu_{4\alpha-rij\Delta T\lambda} = \Sigma_{\alpha^*=1}^{\alpha^*_{max}}[(\alpha_{\alpha^*-rij\Delta T\lambda} - z_{rij\Delta T\lambda}^{A_r/stoch1-1(me)})^4 * p_{\alpha^*-rij\Delta T\lambda}].$$

It is worth noting that Karl Pearson's coefficient of kurtosis as defined above (29) in most practical cases is simply considered, not as an indicator of the forms or trends of changes, but as a measure of the speed of improvement or deterioration in past and future performance by the corresponding parameter [e.g., the smaller the coefficient of kurtosis, the greater is the rapidity of improvement or deterioration (depending on the skewness coefficient value) of a staff member's performance by the corresponding productivity parameter].

3.3. The probability density (actual) ("pd") [23] that the measured performance indicator will not exceed the possible $\alpha_{\alpha^*-rij\Delta T\lambda}$ value:

$$z_{\alpha^*-rij\Delta T\lambda}^{A_r/stoch3-3(pd)} = p(\alpha_{rij\Delta T\lambda} \le \alpha_{\alpha^*-rij\Delta T\lambda}) = \Sigma_{i=1}^{\alpha^*} \frac{k_{i-rij\Delta T\lambda}}{\tau_{max}}. \quad (30)$$

This probability density (actual) $z_{\alpha^*-rij\Delta T\lambda}^{A_r/stoch3-3(pd)}$ (30) can be represented as the reduced probability density ("rpd"). This quantity can be reduced to the mathematical expectation $z_{rij\Delta T\lambda}^{A_r/stoch1-1(me)}$ (25), i.e., as the probability that the first fraction in (31) [the measured performance indicator divided by the mathematical expectation $z_{rij\Delta T\lambda}^{A_r/stoch1-1(me)}$] does not exceed the second fraction in (31) [the possible $\alpha_{\alpha^*-rij\Delta T\lambda}$ value divided by the mathematical expectation $z_{rij\Delta T\lambda}^{A_r/stoch1-1(me)}$]:

$$z_{\alpha^*-rij\Delta T\lambda}^{A_r/stoch3-3(rpd)} = p\left(\frac{\alpha_{rij\Delta T\lambda}}{z_{rij\Delta T\lambda}^{A_r/stoch1-1(me)}} \le \frac{\alpha_{\alpha^*-rij\Delta T\lambda}}{z_{rij\Delta T\lambda}^{A_r/stoch1-1(me)}}\right) = \Sigma_{i=1}^{\alpha^*} \frac{k_{i-rij\Delta T\lambda}}{\tau_{max}}. \quad (31)$$

Note that the probability density (actual) expressed in Eq. (30) and the reduced probability density expressed in Eq. (31) of the **FTC-S** subgroup in practice are considered the same: as the actual and reduced probabilities of relevant staff reaching certain performance values in future periods, as measured by the corresponding productivity parameter.

3.4. The confidence interval (absolute) ("ci") [23], within which, with a given confidence probability ("cp") $p_{\alpha-rij\Delta T\lambda}^{cp}$ and correspondingly a given confidence level ("cl") $p_{\alpha-rij\Delta T\lambda}^{cl}$ for the time elapsed τ_1, τ_2, ..., τ_{max}, the mathematical expectation $z_{rij\Delta T\lambda}^{A_r/stoch1-1(me)}$ has been evaluated (or in the future $\tau_{max+nexttimes}$, the value $\alpha_{rij\tau_{max+next\,times}\Delta T\lambda}$ of the measured performance indicator **r** should have been forecast):

$$z_{\alpha^*-rij\Delta T\lambda}^{A_r/stoch3-4(ci)} = \frac{z_{rij\Delta T\lambda}^{A_r/stoch2-1(sd)}}{\sqrt{\tau_{max}}} * p_{\alpha-rij\Delta T\lambda}^{cl}. \quad (32)$$

The specified confidence interval (absolute) to estimate $z_{\alpha^*-rij\Delta T\lambda}^{A_r/stoch3-4(ci)}$ (32) can be represented as the reduced confidence interval (rci), which is reduced to the mathematical expectation $\mathbf{z}_{\mathbf{rij}\Delta\mathbf{T}\lambda}^{\mathbf{A_r/stoch1-1(me)}}$ (25), i.e., as the ratio of the $\mathbf{z}_{\alpha^*-\mathbf{rij}\Delta\mathbf{T}\lambda}^{\mathbf{A_r/stoch3-4(ci)}}$ estimated confidence interval divided by the mathematical expectation $\mathbf{z}_{\mathbf{rij}\Delta\mathbf{T}\lambda}^{\mathbf{A_r/stoch1-1(me)}}$ and multiplied by 100%:

$$\mathbf{z}_{\alpha^*-\mathbf{rij}\Delta\mathbf{T}\lambda}^{\mathbf{A_r/stoch3-4(rci)}} = \frac{\mathbf{z}_{\alpha^*-\mathbf{rij}\Delta\mathbf{T}\lambda}^{\mathbf{A_r/stoch3-4(ci)}}}{\mathbf{z}_{\mathbf{rij}\Delta\mathbf{T}\lambda}^{\mathbf{A_r/stoch1-1(me)}}} * \mathbf{100\%} \cdot \qquad (33)$$

Attention should be paid to the fact that the absolute and reduced confidence intervals [Eqs. (32) and (33)] of the **FTC-S** subgroup, as defined above, are in most practical cases simply considered, not as indicators of the forms or trends of changes, but rather as measures of the estimation accuracy of the mathematical expectation of the relevant past and future performance parameters of staff members. For example, the smaller the confidence interval, the more accurate is the staff performance indicator assessment according to the given confidence probability.

Chapter 4

Cross-Comparative Analysis of Deterministic and Stochastic Approaches to Staff Performance Evaluation and Forecasting

To perform a comparative analysis of deterministic and stochastic staff performance evaluation and forecasting, the first step is to determine the key cross-comparison indicators (indices and parameters) that will make it possible to select among all staff performance evaluation and forecasting indicators those that most fully and accurately characterize the effectiveness of staff's previous activities and forecast their future effectiveness.

The following main cross-comparison indicators for evaluating and forecasting staff performance can be used as a set of such indicators:

1. **PF** indicator (past-future) the possibility of using indicators to evaluate and forecast past and/or future staff performance. An appropriate strategy would be to consider the indicators for assessing past performance as indicators of staff performance results and to consider the indicators for assessing expected future performance as indicators of anticipated staff performance.

2. **IC** (informational content) is a measure of the informativeness of an indicator for evaluating and forecasting staff performance.

The indicator **IC** should characterize the indicator's assessment capabilities in determining the mean (**IC mean**) and/or dispersion (**IC-dispersion**) and/or change trends (**IC-change**) of staff performance indicators and should also characterize the indicator's capabilities for comparative performance determination for different sets of employees working under equal (**IC-e-condition**) and/or different (**IC-d-condition**) conditions in the same (**IC-e-direction**) and/or different (**IC-d-direction**) areas of activity.

3. The **AC** (accuracy) indicator reflects the indicators' accuracy in evaluating and forecasting staff performance.

The specified **AC** indicator should characterize the accuracy of the calculated measurement indicator (under the compulsory preconditioning assumption that any instrumental and/or subjective/personal errors are excluded) as the absolute accuracy **AC-A** and/or as a relative parametric accuracy **AC-RC**, depending on the accuracy of the components used to calculate the measuring indicator, and/or as a relative methodological accuracy **AC-RM**, which depends on the accuracy of the methods used to calculate the measured parameter.

4. The **SM** (simplicity) indicator refers to the ease of obtaining indicators for evaluating and forecasting staff performance.

Note that direct mathematical calculations of a particular indicator for evaluating and forecasting staff performance will be, on the one hand, of different levels of complexity, but that on the other hand, these computations will be carried out (that is, all these difficulties will be overcome) by hardware and software, and that therefore these difficulties should not be noticeable to the enterprises' representatives who are responsible for calculating the corresponding indicators. Based on this discussion, the level of complexity of the mathematical calculations of indicators for staff performance evaluation and forecasting should not be accounted for using the **SM** indicator.

The **SM** indicator characterizes the degree of complexity (simplicity) of collecting (registering) measured values for the performance indicator **r** for each **i**-th employee of the corresponding **j**-th enterprise in the measurement time interval $\Delta\mathbf{T}$. The first degree of complexity (the easiest) is **SM-month**, when the measured values are registered once a month. The second degree of complexity (complex) is **SM-week**, which involves registering measured values once a week. The third degree of complexity (more complex) is **SM-day**, which involves registering measured values once a day, and the fourth degree of complexity (even more complex) is **SM-hour**, with which the measured values are registered once an hour.

5. **CS** (cost) refers to the cost of obtaining indicators for evaluating and forecasting staff performance.

The specified **CS** indicator should characterize the costs of calculating the measured indicator per estimated employee as **CS-staff** costs from the total monthly expenses to maintain this estimated employee. The **CS** indicator should be proportional to the time taken by this employee to determine and register the measured values of the employee's own performance indicator **r**, as **CS-specialist** costs from the estimators' total monthly costs, in proportion to the time that these estimators are involved in assessing, registering, and processing the measured values and calculating the performance indicator **r** for the estimated employee, and as

CS-hard&soft market costs for purchasing and using hardware and software to estimate, register, and process the measured values with subsequent calculation of the estimated employee's performance indicator **r**.

CS-staff (**CS-staff**, which is the monthly expenses $Q^{CS-staff}_{rij-month}$ to measure by employee the performance indicator **r** for the **i**-th employee of the **j**-th enterprise) can be represented as the product $Q^{CS-staff}_{rij-month}$ of multiplying the total monthly expenses $Q^{Total}_{ij-month}$ for the maintenance of the **i**-th estimated specialist by the ratio of the time $T^{CS-staff}_{rij-month}$ when this **i**-th estimated employee is involved in the evaluation and forecasting procedure up to assessing and registering the measured values of the employee's own performance indicator **r**, divided by this **i**-th estimated employee's total work duration $T^{Total}_{ij-month}$ for one month:

$$CS-staff = Q^{CS-staff}_{rij-month} = Q^{Total}_{ij-month} * \frac{T^{CS-staff}_{rij-month}}{T^{Total}_{ij-month}} .$$

In turn, the monthly expenses **CS-specialists** (where **CS-specialists** is the monthly expenses $Q^{CS-specialists}_{rij-month}$ incurred by estimators to measure the performance indicator **r** for the **i**-th employee of the **j**-th enterprise) can be represented as the product $Q^{CS-specialists}_{rij-month}$ of multiplying the total monthly expenses $Q^{Total}_{Specialistsj-month}$ for the maintenance of the estimators by the ratio of the time $T^{CS-specialists}_{rij-month}$ spent on working with those estimators to measure, register, and process the measured values and the performance indicator **r** calculated for the **i**-th estimated employee, divided by the total duration $T^{Total}_{Specialistsj-month}$ of the estimators' work for one month:

$$CS-specialists = Q^{CS-specialists}_{rij-month} = Q^{Total}_{Specialistsj-month} * \frac{T^{CS-specialists}_{rij-month}}{T^{Total}_{Specialistsj-month}} .$$

Of course, the list of key indicators **PF**, **IC**, **AC**, **SM**, and **CS** can lay no claim to absolute completeness—whichever personnel study may be carried out can use, apart from the indicators above, other additional indicators of cross-comparison parameters for evaluating and forecasting the performance of various types of staff.

For greater clarity of cross-comparison, all analyzed deterministic and stochastic indicators for staff performance evaluation and forecasting (see Chapters 2 and 3 respectively) and the appropriate values of the comparative indicators **PF**, **IC**, **AC**, **SM**, and **CS** are summarized in Table 3.

Table 3
Cross-comparison table of the deterministic and stochastic indicators of staff performance evaluation and forecasting using the comparative indicators **PF**, **IC**, **AC**, **SM**, and **CS**.

Staff performance evaluation and forecasting indicator	Comparative indicator				
	PF ("past-future" indicator)	**IC** (information content indicator)	**AC** (accuracy indicator)	**SM** (simplicity indicator)	**CS** (cost indicator)
1. Deterministic indicators for staff performance evaluation and forecasting					
1.1. Subgroup of MV-D mean (measures of central tendency) of measured values					
Arithmetic mean $Z_{rij\Delta T\lambda}^{A_r/det1-1(am)}$	Evaluating past staff performance	IC-mean, IC-e-condition, IC-e-direction	AC-A	SM-month or SM-week or SM-day or SM-hour, etc.	CS-staff + +CS-specialists+ +CS-hard&soft
Weighted mean $Z_{rij\Delta T\lambda}^{A_r/det1-2(wm)}$	Evaluating past staff performance	IC-mean, IC-e-condition, IC-d-condition, IC-e-direction	AC-RC (accuracy depending on the accuracy of determining the applied weight coefficients ω)	SM-month or SM-week or SM-day or SM-hour, etc.	CS-staff + +CS-specialists+ +CS-hard&soft
Geometric mean $Z_{rij\Delta T\lambda}^{A_r/det1-3(gm)}$	Evaluating past staff performance	IC-mean, IC-e-condition, IC-e-direction	AC-A	SM-month or SM-week or SM-day or SM-hour, etc.	CS-staff + +CS-specialists+ +CS-hard&soft
Harmonic mean $Z_{rij\Delta T\lambda}^{A_r/det1-4(hm)}$	Evaluating past staff performance	IC-mean, IC-e-condition, IC-e-direction	AC-A	SM-month or SM-week or SM-day or SM-hour, etc.	CS-staff + +CS-specialists+ +CS-hard&soft
Median $Z_{rij\Delta T\lambda}^{A_r/det1-5(med-odd)}$ $Z_{rij\Delta T\lambda}^{A_r/det1-5(med-even)}$	Evaluating past staff performance	IC-mean, IC-e-condition, IC-e-direction	AC-A	SM-month or SM-week or SM-day or SM-hour, etc.	CS-staff + +CS-specialists+ +CS-hard&soft
Mode $Z_{rij\Delta T\lambda}^{A_r/det1-6(mod-1)}$ $Z_{rij\Delta T\lambda}^{A_r/det1-6(mod-n)}$	Evaluating past staff performance	IC-mean, IC-e-condition, IC-e-direction	AC-A	SM-month or SM-week or SM-day or SM-hour, etc.	CS-staff + +CS-specialists+ +CS-hard&soft

1.2. Subgroup of VDR-D variation and deviation ranges (measures of spread) of measured values					
Mean absolute deviation $Z_{rij\Delta T\lambda}^{A_r/det2-1(mad)}$	Evaluating past staff performance	IC-dispersion, IC-e-condition, IC-e-direction	AC-A	SM-month or SM-week or SM-day or SM-hour, etc.	CS-staff + +CS-specialists+ +CS-hard&soft
Mean relative deviation $Z_{rij\Delta T\lambda}^{A_r/det2-2(mrd)}$	Evaluating past staff performance	IC-dispersion, IC-e-condition, IC-d-condition, IC-e-direction, IC-d-direction	AC-A	SM-month or SM-week or SM-day or SM-hour, etc.	CS-staff + +CS-specialists+ +CS-hard&soft
Deviation range $Z_{rij\Delta T\lambda}^{A_r/det2-3(ran)}$	Evaluating past staff performance	IC-dispersion, IC-e-condition, IC-e-direction	AC-A	SM-month or SM-week or SM-day or SM-hour, etc.	CS-staff + +CS-specialists+ +CS-hard&soft
1.3. Subgroup of FTC-D forms and trends of changes (probability measures) of measured values					
Linear trend-line approximation function $F_{lin}(\alpha_{rij\tau\Delta T\lambda})$	Forecasting future staff performance	IC-change, IC-e-condition, IC-e-direction	AC-RM (accuracy depending on the accuracy of the least squares method)	SM-month or SM-week or SM-day or SM-hour, etc.	CS-staff + +CS-specialists+ +CS-hard&soft
Polynomial trend-line approximation function $F_{pol}(\alpha_{rij\tau\Delta T\lambda})$	Forecasting future staff performance	IC-change, IC-e-condition, IC-e-direction	AC-RM (accuracy depending on the accuracy of the least squares method)	SM-month or SM-week or SM-day or SM-hour, etc.	CS-staff + +CS-specialists+ +CS-hard&soft
Exponential trend-line approximation function $F_e(\alpha_{rij\tau\Delta T\lambda})$	Forecasting future staff performance	IC-change, IC-e-condition, IC-e-direction	AC-RM (accuracy depending on the accuracy of the least squares method)	SM-month or SM-week or SM-day or SM-hour, etc.	CS-staff + +CS-specialists+ +CS-hard&soft
Power trend-line approximation function $F_{pow}(\alpha_{rij\tau\Delta T\lambda})$	Forecasting future staff performance	IC-change, IC-e-condition, IC-e-direction	AC-RM (accuracy depending on the accuracy of the least squares method)	SM-month or SM-week or SM-day or SM-hour, etc.	CS-staff + +CS-specialists+ +CS-hard&soft

Logarithmic trend-line approximation function $\mathbf{F_{ln}(\alpha_{rij\tau\Delta T\lambda})}$	Forecasting future staff performance	IC-change, IC-e-condition, IC-e-direction	AC-RM (accuracy depending on the accuracy of the least squares method)	SM-month or SM-week or SM-day or SM-hour, etc.	CS-staff + +CS-specialists+ +CS-hard&soft
Simple moving average smoothing $\mathbf{\hat{z}_{rij\tau_{max+1}\Delta T\lambda}}$	Forecasting future staff performance	IC-change, IC-e-condition, IC-e-direction	AC-RM (accuracy depending on the accuracy of the method of estimating error and confidence intervals using Student's distribution)	SM-month or SM-week or SM-day or SM-hour, etc.	CS-staff + +CS-specialists+ +CS-hard&soft
Weighted moving average smoothing $\mathbf{\hat{\hat{z}}_{rij\tau_{max+1}\Delta T\lambda}}$	Forecasting future staff performance	IC-change, IC-e-condition, IC-d-condition, IC-e-direction	AC-RC (accuracy depending on the accuracy of determining the applied weight coefficients $\boldsymbol{\omega}$) AC-RM (accuracy depending on the accuracy of the method of estimating error and confidence intervals using Student's distribution)	SM-month or SM-week or SM-day or SM-hour, etc.	CS-staff + +CS-specialists+ +CS-hard&soft
Exponential approximation $\mathbf{\tilde{z}_{rij\tau_{max+1}\Delta T\lambda}}$	Forecasting future staff performance	IC-change, IC-e-condition, IC-e-direction	AC-RC (accuracy depending on the accuracy of determining the applied smoothing constant θ) AC-RM (accuracy depending on the accuracy of the method of estimating error and confidence intervals using Student's distribution)	SM-month or SM-week or SM-day or SM-hour, etc.	CS-staff + +CS-specialists+ +CS-hard&soft

2. Stochastic indicators for staff performance evaluation and forecasting					
2.1. Subgroup of MV-S mean (measures of central tendency) of the measured values					
Mathematical expectation $Z_{rij\Delta T\lambda}^{A_r/stoch1-1(me)}$	Evaluating and forecasting past and future staff performance	IC-mean, IC-e-condition, IC-e-direction	AC-RM (accuracy depending on the accuracy of the method of estimating the confidence interval with a given confidence level using the standard normal distribution)	SM-month or SM-week or SM-day or SM-hour, etc.	CS-staff + +CS-specialists+ +CS-hard&soft
2.2. Subgroup of VDR-S variation and deviation ranges (measures of spread) of measured values					
Standard deviation $Z_{rij\Delta T\lambda}^{A_r/stoch2-1(sd)}$	Evaluating and forecasting past and future staff performance	IC-dispersion, IC-e-condition, IC-e-direction	AC-A	SM-month or SM-week or SM-day or SM-hour, etc.	CS-staff + +CS-specialists+ +CS-hard&soft
Coefficient of variation $Z_{rij\Delta T\lambda}^{A_r/stoch2-2(cv)}$	Evaluating and forecasting past and future staff performance	IC-dispersion, IC-e-condition, IC-d-condition, IC-e-direction, IC-d-direction	AC-A	SM-month or SM-week or SM-day or SM-hour, etc.	CS-staff + +CS-specialists+ +CS-hard&soft
2.3. Subgroup of FTC-S forms and trends of changes (probability measures) of measured values					
Coefficient of skewness $Z_{rij\Delta T\lambda}^{A_r/stoch3-1(cs)}$	Evaluating and forecasting past and future staff performance	IC-change, IC-e-condition, IC-d-condition, IC-e-direction, IC-d-direction	AC-A	SM-month or SM-week or SM-day or SM-hour, etc.	CS-staff + +CS-specialists+ +CS-hard&soft

Coefficient of kurtosis $Z_{rij\Delta T\lambda}^{A_r/stoch3-2(ck)}$	Evaluating and forecasting past and future staff performance	IC-change, IC-e-condition, IC-d-condition, IC-e-direction, IC-d-direction	AC-A	SM-month or SM-week or SM-day or SM-hour, etc.	CS-staff + +CS-specialists+ +CS-hard&soft
Probability density (actual) $Z_{\alpha^*-rij\Delta T\lambda}^{A_r/stoch3-3(pd)}$	Evaluating and forecasting past and future staff performance	IC-change, IC-e-condition, IC-e-direction	AC-A	SM-month or SM-week or SM-day or SM-hour, etc.	CS-staff + +CS-specialists+ +CS-hard&soft
Probability density (reduced) $Z_{\alpha^*-rij\Delta T\lambda}^{A_r/stoch3-3(rpd)}$	Evaluating and forecasting past and future staff performance	IC-change, IC-e-condition, IC-d-condition, IC-e-direction, IC-d-direction	AC-A	SM-month or SM-week or SM-day or SM-hour, etc.	CS-staff + +CS-specialists+ +CS-hard&soft
Confidence interval (absolute) of the assessment $Z_{\alpha^*-rij\Delta T\lambda}^{A_r/stoch3-4(ci)}$	Evaluating and forecasting past and future staff performance	IC-change, IC-e-condition, IC-e-direction	AC-A	SM-month or SM-week or SM-day or SM-hour, etc.	CS-staff + +CS-specialists+ +CS-hard&soft
Confidence interval (reduced) of the estimate $Z_{\alpha^*-rij\Delta T\lambda}^{A_r/stoch3-4(rci)}$	Evaluating and forecasting past and future staff performance	IC-change, IC-e-condition, IC-d-condition, IC-e-direction, IC-d-direction	AC-A	SM-month or SM-week or SM-day or SM-hour, etc.	CS-staff + +CS-specialists+ +CS-hard&soft

As follows from the data analysis in Table 3, all the deterministic and stochastic indicators used to measure staff performance are almost identical for the comparative indicators of simplicity **SM** and cost **CS**. This is the case because the degree of complexity in collecting measured performance values is determined solely by the number of measured values registered (regardless of the staff performance indicator), and the cost of calculating the performance indicators is determined solely in proportion to the monthly expenditure for the estimated employee and the expert estimator (which, in turn, is proportional to the number of measured values registered) and the market costs of the relevant hardware and software for determining, capturing, and processing the measured values (which are almost independent of the staff performance evaluation and forecasting indicator). The specified identity of the simplicity and cost indicators is based on the fact that, first, the total amount of initial data (that is, the number of measured values registered) is the same for different staff measurement indicators, and therefore, second, the market costs for the corresponding hardware and software to determine, register, and process the measured values are almost the same, despite relatively minor mathematical differences in calculating the staff performance evaluation and forecasting indicators.

Based on the above, among all indicators for comparing the previously mentioned deterministic and stochastic performance evaluation and forecasting indicators, three critical comparative indicators should remain:

– **PF**, an indicator of the possibility of using indicators to evaluate and forecast past and future staff performance;

– **IC**, an indicator of the information content of staff performance evaluation and forecasting indicators;

– **AC**, an indicator of the accuracy of staff performance evaluation and forecasting indicators.

The following discussion presents a more detailed comparison of deterministic and stochastic staff performance evaluation and forecasting indicators using the comparative indicators **PF**, **IC**, and **AC**.

According to the data analysis presented in Table 3, all the deterministic and stochastic staff performance indicators associated with the **PF** indicator (“past-future”) can be clearly divided into the following three groups:

• A group of **PST** indicators for evaluating past staff performance (evaluating the results of staff activities), which includes all deterministic indicators of the **MV-D** subgroup of means and the **VDR-D** subgroup of variation and deviation ranges for the measured values:

∘ the arithmetic mean $z_{rij\Delta T\lambda}^{A_r/det1-1(am)}$ in the **MV-D** subgroup;

∘ the weighted mean $z_{rij\Delta T\lambda}^{A_r/det1-2(wm)}$ in the **MV-D** subgroup;

∘ the geometric mean $z_{rij\Delta T\lambda}^{A_r/det1-3(gm)}$ in the **MV-D** subgroup;

∘ the harmonic mean $z_{rij\Delta T\lambda}^{A_r/det1-4(hm)}$ in the **MV-D** subgroup;

∘ the median $z_{rij\Delta T\lambda}^{A_r/det1-5(med-odd)}$ (for an odd number of measured values) or $z_{rij\Delta T\lambda}^{A_r/det1-5(med-even)}$ (for an even number of measured values) in the **MV-D** subgroup;

∘ the mode $z_{rij\Delta T\lambda}^{A_r/det1-6(mod-1)}$ (for a single-modal finite sequence of measured values) or $z_{rij\Delta T\lambda}^{A_r/det1-6(mod-n)}$ [for a multi-modal (n-modal) finite sequence of measured values] in the **MV-D** subgroup;

∘ the mean absolute deviation $z_{rij\Delta T\lambda}^{A_r/det2-1(mad)}$ in the **VDR-D** subgroup;

∘ the mean relative deviation $z_{rij\Delta T\lambda}^{A_r/det2-2(mrd)}$ in the **VDR-D** subgroup;

∘ the deviation range $z_{rij\Delta T\lambda}^{A_r/det2-3(ran)}$ in the **VDR-D** subgroup.

• A group of **FTR** indicators for forecasting future staff performance (forecasting the results of staff activities), which includes all parametric and non-parametric deterministic indicators of the **FTC-D** subgroup of the forms or trends of changes in measured values. These are calculated using the following functions and methods:

∘ the parametric linear trend-line approximation function $F_{lin}(\alpha_{rij\tau\Delta T\lambda})$, which determines the parametric indicator for the **FTC-D** subgroup at the corresponding future times $\tau_{max+1}, \tau_{max+2}, \ldots$;

∘ the parametric polynomial trend-line approximation function $F_{pol}(\alpha_{rij\tau\Delta T\lambda})$, which determines the parametric indicator for the **FTC-D** subgroup at the corresponding future times $\tau_{max+1}, \tau_{max+2}, \ldots$;

∘ the parametric exponential trend-line approximation function $F_{e}(\alpha_{rij\tau\Delta T\lambda})$, which determines the parametric indicator for the **FTC-D** subgroup at the corresponding future times $\tau_{max+1}, \tau_{max+2}, \ldots$;

∘ the parametric power-law trend-line approximation function $F_{pow}(\alpha_{rij\tau\Delta T\lambda})$, which determines the parametric indicator for the **FTC-D** subgroup at the corresponding future times $\tau_{max+1}, \tau_{max+2}, \ldots$;

∘ the parametric logarithmic trend-line approximation function $F_{ln}(\alpha_{rij\tau\Delta T\lambda})$, which determines the parametric indicator for the **FTC-D** subgroup at the corresponding future times $\tau_{max+1}, \tau_{max+2}, \ldots$;

∘ the non-parametric simple moving average smoothing method, which determines the non-parametric indicator $\hat{z}_{rij\tau_{max+1}\Delta T\lambda}$ for the **FTC-D** subgroup;

∘ the non-parametric weighted moving average smoothing method, which determines the non-parametric indicator $\hat{\hat{\mathbf{z}}}_{\mathbf{rij\tau_{max+1}\Delta T\lambda}}$ for the **FTC-D** subgroup;

∘ the non-parametric exponential smoothing method, which determines the non-parametric indicator $\tilde{\mathbf{z}}_{\mathbf{rij\tau_{max+1}\Delta T\lambda}}$ for the **FTC-D** subgroup.

• The **PST&FTR** group of indicators to assess performance simultaneously for the past (evaluating staff performance results) and the future (forecasting staff performance). This group includes all the stochastic indicators in the **MV-S** subgroup of means, the **VDR-S** subgroup of variation and deviation ranges, and the **FTC-S** subgroup of the forms or trends of changes in the measured values:

∘ the mathematical expectation $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/stoch1-1(me)}}$, which is an indicator in the **MV-S** subgroup;

∘ the standard deviation $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/stoch2-1(sd)}}$, which is an indicator in the **VDR-S** subgroup;

∘ the coefficient of variation $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/stoch2-2(cv)}}$, which is an indicator in the **VDR-S** subgroup;

∘ Karl Pearson's skewness coefficient $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/stoch3-1(cs)}}$, which is an indicator in the **FTC-S** subgroup;

∘ Karl Pearson's kurtosis coefficient $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/stoch3-2(ck)}}$, which is an indicator in the **FTC-S** subgroup;

∘ the probability density (actual) $\mathbf{z}_{\mathbf{\alpha^*-rij\Delta T\lambda}}^{\mathbf{A_r/stoch3-3(pd)}}$, which is an indicator in the **FTC-S** subgroup;

∘ the probability density (reduced) $\mathbf{z}_{\mathbf{\alpha^*-rij\Delta T\lambda}}^{\mathbf{A_r/stoch3-3(rpd)}}$, which is an indicator in the **FTC-S** subgroup;

∘ the confidence interval (absolute) $\mathbf{z}_{\mathbf{\alpha^*-rij\Delta T\lambda}}^{\mathbf{A_r/stoch3-4(ci)}}$, which is an indicator in the **FTC-S** subgroup;

∘ the confidence interval (reduced) $\mathbf{z}_{\mathbf{\alpha^*-rij\Delta T\lambda}}^{\mathbf{A_r/stoch3-4(rci)}}$, which is an indicator in the **FTC-S** subgroup.

In turn, according to the data analysis presented in Table 3, all the given deterministic and stochastic staff performance indicators for the **IC** indicator of information content can be conditionally divided into subgroups within each of the three groups listed above (**PST**, **FTR**, and **PST&FTR**) in accordance with the "past-future" **PF** indicator:

First, all deterministic staff performance indicators in the **PST** group, based on the **IC** information content assessment indicator, can be clearly divided into the following four subgroups:

• The basic **IC-BE** subgroup of the **PST** group, which provides a minimum information content for evaluating staff performance. It is a subgroup of average (unweighted) indicators of measured values that includes all unweighted deterministic indicators from the **MV-D** subgroup:

- the arithmetic mean $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/det1-1(am)}}$;
- the geometric mean $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/det1-3(gm)}}$;
- the harmonic mean $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/det1-4(hm)}}$;
- the median $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/det1-5(med-odd)}}$ (for an odd number of measured values);
- the median $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/det1-5(med-even)}}$ (for an even number of measured values);
- the mode $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/det1-6(mod-1)}}$ (for a single-modal finite sequence of measured values);
- the mode $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/det1-6(mod-n)}}$ (for a multi-modal finite sequence of measured values).

The **IC-BE** subgroup of the **PST** group should be considered as a subset providing a basic information content because the indicators belonging to this group are basic components of other groups of measured value evaluation indicators according to the **IC** (information content assessment) indicator. At the same time, the **IC-BE** subgroup should be considered as a subset providing the minimum information content for an estimate because, first, this subgroup can estimate only the average of the measured values, and second, using this subset, one can carry out a cross-comparative performance evaluation analysis only for employees who work under the same conditions (**IC-e-condition**) in the same areas of activity (**IC-e-direction**).

- **IC-WE** subgroup of the **PST** group provides a weighted information level for staff performance evaluation. It is a subgroup of weighted mean indicators for measured values and includes the deterministic weighted indicator from the **MV-D** subgroup:
  - the weighted mean value $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/det1-2(wm)}}$.

The **IC-WE** subgroup of the **PST** group should be considered as a subset of a potentially (with accurate determination of the weight coefficients $\boldsymbol{\omega}$ used) higher information level than the **IC-BE** subgroup because using this subset, one can carry out a cross-comparative performance evaluation analysis of employees who work in the same areas of activity (**IC-e-direction**), but under either the same (**IC-e-condition**) or different (**IC-d-condition**) conditions.

- The combined **IC-DAE** subgroup of the **PST** group provides an absolute range information content for staff performance evaluation. This subgroup cumulatively combines the basic **IC-BE** subgroup considered earlier and the separate additional components of the subgroup of indicators of variation and deviation ranges for the measured values, including the absolute deterministic indicators in the **VDR-D** subgroup:

- the mean absolute deviation $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/det2-1(mad)}}$;
- the deviation range $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/det2-3(ran)}}$.

The **IC-DAE** subgroup of the **PST** group should be considered as a subgroup with a higher level of information content than **IC-BE**, which also belongs to the **PST** group. Using this subgroup, a cross-comparative employee performance analysis can be carried out not only by mean indicators, but also by indicators of absolute deviation (that is, by indicators of absolute stability of employee activity) under the same conditions (**IC-e-condition**) and in the same areas of activity (**IC-e-direction**).

• The combined **IC-DRE** subgroup of the **PST** group provides indicators of relative range information content for staff performance evaluation. **IC-DRE** is a subgroup that collectively unifies the combined **IC-DAE** subgroup of the **PST** group considered earlier and an additional separate component of the subgroup of variation and deviation ranges for measured values, including the relative deterministic indicator of the **VDR-D** subgroup:

- the mean relative deviation $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/det2-2(mrd)}}$.

The **IC-DRE** subgroup of the **PST** group should be considered as a subgroup with a level of evaluation information content that is even higher than that specific to the **IC-DAE** subgroup of the **PST** group. This is the case because using this subgroup, a cross-comparative employee performance analysis can be carried out, not only by average indicators and the level of absolute deviations, but also by the level of relative variance (i.e., the level of relative stability of employee activity) for staff working under the same (**IC-e-condition**) or different (**IC-d-condition**) conditions, as well as working in the same (**IC-e-direction**) or different (**IC-d-direction**) activity areas.

Second, all deterministic staff performance indicators in the **FTR** group based on the **IC** information content prediction indicator can be clearly divided into the following two subgroups:

• The **IC-SF** subgroup of the **FTR** group: this subgroup provides a standard level of information content for staff performance forecasting. It includes parametric and non-parametric unweighted deterministic indicators of the forms or trends of changes of the measured values. This subgroup contains all unweighted deterministic indicators from the **FTC-D** subgroup that are calculated using the following functions and methods:

- the parametric linear trend-line approximation function $\mathbf{F_{lin}(\alpha_{rij\tau\Delta T\lambda})}$;
- the parametric polynomial trend-line approximation function $\mathbf{F_{pol}(\alpha_{rij\tau\Delta T\lambda})}$;

- the parametric exponential trend-line approximation function $\mathbf{F_e(\alpha_{rij\tau\Delta T\lambda})}$;
- the parametric power-law trend-line approximation function $\mathbf{F_{pow}(\alpha_{rij\tau\Delta T\lambda})}$;
- the parametric logarithmic trend-line approximation function $\mathbf{F_{ln}(\alpha_{rij\tau\Delta T\lambda})}$;
- the non-parametric simple moving average smoothing method;
- the non-parametric exponential smoothing method.

The **IC-SF** subgroup of the **FTR** group should be considered as a subgroup with the lowest level of information content forecasting indicators. This is the case because using this subgroup, a cross-comparative performance forecasting analysis can be carried out only for employees who work under the same conditions (**IC-e-condition**) and in the same areas of activity (**IC-e-direction**).

- The **IC-WF** subgroup of the **FTR** group offers a specific weighted level of staff performance forecasting information content. This is the subgroup of parametric deterministic weighted indicators of the forms or trends of changes of measured values that applies to the deterministic weighted indicator of the **FTC-D** subgroup and is calculated using the following method:
  - the non-parametric weighted moving average smoothing method.

The **IC-WF** subgroup of the **FTR** group should be considered as a subset with a potentially (with accurate estimation of the weight coefficient $\boldsymbol{\omega}$ used) higher information level than the **IC-SF** subgroup of the **PST** group. Using this subset, one can carry out a cross-comparative performance forecasting analysis of employees who work in the same areas of activity (**IC-e-direction**), but under either the same (**IC-e-condition**) or different (**IC-d-condition**) conditions.

Third, all stochastic staff performance indicators of the **PST&FTR** group based on the **IC** indicator can be clearly divided by their information content for evaluation and forecasting into the following four subgroups:

- The basic **IC-BEF** subgroup of the **PST&FTR** group has the minimum information content for evaluating and forecasting staff performance. It is a subgroup of mean indicators for measured values and includes the stochastic indicator of the **MV-S** subgroup:
  - the mathematical expectation $\mathbf{z_{rij\Delta T\lambda}^{A_r/stoch1-1(me)}}$.

The **IC-BEF** subgroup of the **PST&FTR** group should be considered as a subset of the basic level of evaluation and forecasting information content because this subgroup indicator constitutes the basic component for other subgroups of measured value indicators based on the **IC** indicator of evaluation and forecasting information content. At the same time, the **IC-BEF** subgroup should be considered as a subset with the minimum information level for estimation because, first, this subgroup can estimate only the mean of the measured values, and second, one can carry out a cross-comparative performance evaluation and forecasting analysis using this subgroup only for employees who work under the same conditions (**IC-e-condition**) and in the same areas of activity (**IC-e-direction**).

• The combined **IC-DAEF** subgroup of the **PST&FTR** group provides an absolute range information content for staff performance evaluation and forecasting. This subgroup cumulatively combines the basic **IC-BEF** subgroup of the **PST&FTR** group considered earlier and a separate component of the subgroup of variation and deviation ranges of the measured values, to which the absolute stochastic indicator of the **VDR-S** subgroup belongs:

∘ standard deviation $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/stoch2-1(sd)}}$.

The **IC-DAEF** subgroup of the **PST&FTR** group should be considered as a subgroup with a higher evaluation and forecasting information content than that associated with **IC-BEF**, which also belongs to the **PST&FTR** group. Using this subgroup, a cross-comparative employee performance analysis can be carried out, not only by mean indicators, but also by indicators of absolute deviation (that is, by the absolute stability of employee activity) under the same conditions (**IC-e-condition**) and in the same areas of activity (**IC-e-direction**).

• The combined **IC-DREF** subgroup of the **PST&FTR** group, which contains information about absolute and relative ranges for staff performance evaluation and forecasting, is a subgroup that collectively unifies the combined **IC-DAEF** subgroup of the **PST&FTR** group considered earlier and an additional separate component of the subgroup of variation and deviation ranges of the measured values, including the relative stochastic indicator of the **VDR-S** subgroup:

∘ the coefficient of variation $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/stoch2-2(cv)}}$.

The **IC-DREF** subgroup of the **PST&FTR** group should be considered as having an evaluation and forecasting information content that is even higher than that of the **IC-DAEF** subgroup of the **PST&FTR** group. Using this subgroup, a cross-comparative employee performance evaluation and forecasting analysis can be carried out not only by mean indicators and by absolute deviations, but also by the level of relative deviations (i.e., the level of relative stability of employee activity) for staff working under either the same (**IC-e-condition**) or different (**IC-d-condition**) conditions, as well as in the same (**IC-e-direction**) or different (**IC-d-direction**) activity directions.

- The combined **IC-MAREF** subgroup of the **PST&FTR** group has maximum absolute and relative staff evaluation and forecasting information level. It collectively combines the previously discussed combined **IC-DREF** subgroup of the **PST&FTR** group and in addition a subgroup of stochastic indicators among the forms or trends of changes of measured values, which includes all the stochastic indicators of the **FTC-S** subgroup:
  - Karl Pearson's skewness coefficient $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/stoch3-1(cs)}}$;
  - Karl Pearson's kurtosis coefficient $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/stoch3-2(ck)}}$;
  - the probability density (actual) $\mathbf{z}_{\alpha^*-\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/stoch3-3(pd)}}$;
  - the probability density (reduced) $\mathbf{z}_{\alpha^*-\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/stoch3-3(rpd)}}$;
  - the confidence interval (absolute) $\mathbf{z}_{\alpha^*-\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/stoch3-4(ci)}}$;
  - the confidence interval (reduced) $\mathbf{z}_{\alpha^*-\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/stoch3-4(rci)}}$.

The **IC-MAREF** subgroup of the **PST&FTR** group should be considered as the subgroup with the maximum level of information content for evaluation and forecasting compared to all the previously discussed subgroups (evaluation, forecasting, and evaluation and forecasting). Using this subgroup, a comparative employee performance evaluation and forecasting analysis can be carried out, not only on the mean indicators and the level of absolute and relative deviations (level of stability), but also at the levels of absolute and relative indicators of the forms or trends of changes (i.e., trends and levels of absolute and relative prospective performance, probability levels of achieving certain values, and levels of accuracy for evaluation and forecasting means of the measured values) for employees working under the same (**IC-e-condition**) or different (**IC-d-condition**) conditions and in the same (**IC-e-direction**) or different (**IC-d-direction**) activity directions.

Furthermore, according to the data analysis shown in Table 3, all the deterministic and stochastic staff performance indicators presented here can be clearly divided by the **AC** accuracy indicator into the following four groups:

- Group **AC-A**: maximum calculation accuracy of indicators to evaluate and forecast staff performance, including all unweighted deterministic indicators in the **MV-D** subgroup (means) and the **VDR-D** subgroup (variation and deviation ranges) of the measured values, as well as all the stochastic indicators in the **VDR-S** subgroup of variation and deviation ranges and the **FTC-S** subgroup of forms and trends of changes in measured values:

- the arithmetic mean $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/det1-1(am)}}$;
- the geometric mean $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/det1-3(gm)}}$;
- the harmonic mean $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/det1-4(hm)}}$;
- the median $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/det1-5(med-odd)}}$ (for an odd number of measured values);
- the median $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/det1-5(med-even)}}$ (for an even number of measured values);
- the mode $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/det1-6(mod-1)}}$ (for a single-modal finite sequence of measured values);
- the mode $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/det1-6(mod-n)}}$ (for a multi-modal finite sequence of measured values);
- the mean absolute deviation $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/det2-1(mad)}}$;
- the mean relative deviation $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/det2-2(mrd)}}$;
- the deviation range $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/det2-3(ran)}}$;
- the standard deviation $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/stoch2-1(sd)}}$;
- the coefficient of variation $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/stoch2-2(cv)}}$;
- Karl Pearson's skewness coefficient $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/stoch3-1(cs)}}$;
- Karl Pearson's kurtosis coefficient $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/stoch3-2(ck)}}$;
- the probability density (actual) $\mathbf{z}_{\alpha^*-\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/stoch3-3(pd)}}$;
- the probability density (reduced) $\mathbf{z}_{\alpha^*-\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/stoch3-3(rpd)}}$;
- the confidence interval (absolute) $\mathbf{z}_{\alpha^*-\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/stoch3-4(ci)}}$;
- the confidence interval (reduced) $\mathbf{z}_{\alpha^*-\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/stoch3-4(rci)}}$.

The **AC-A** group should be considered as a subgroup with absolute (i.e., maximum) accuracy in calculating indicators for evaluating and forecasting staff performance [under the mandatory conditional assumption that there are no instrumental (hardware/software) or subjective/personal errors].

- The **AC-RC** group has a relative parametric accuracy in calculating staff performance evaluation indicators and includes a weighted deterministic indicator of the **MV-D** subgroup of measured value averages:
  - the weighted mean value $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/det1-2(wm)}}$.

The **AC-RC** group should be considered as a subgroup with a relative accuracy in calculating employee performance indicators, depending on the accuracy of the calculation components of the measuring indicator.

- The **AC-RM** group has a relative methodological accuracy in calculating the indicators to forecast staff performance, which includes all (except those obtained using the non-parametric weighted average method and the non-parametric exponential approximation method) deterministic indicators of the **FTC-D** subgroup (forms and trends of changes), as well as the stochastic indicator of the **MV-S** subgroup of mean measured values:

- the parameter calculated using the parametric linear trend-line approximation function $F_{lin}(\alpha_{rij\tau\Delta T\lambda})$;
- the parameter calculated using the parametric polynomial trend-line approximation function $F_{pol}(\alpha_{rij\tau\Delta T\lambda})$;
- the parameter calculated using the parametric exponential trend-line approximation function $F_{e}(\alpha_{rij\tau\Delta T\lambda})$;
- the parameter calculated using the parametric power-law trend-line approximation function $F_{pow}(\alpha_{rij\tau\Delta T\lambda})$;
- the parameter calculated using the parametric logarithmic trend-line approximation function $F_{ln}(\alpha_{rij\tau\Delta T\lambda})$;
- the parameter calculated using the non-parametric simple moving average smoothing method;
- the mathematical expectation $Z_{rij\Delta T\lambda}^{A_r/stoch1-1(me)}$.

The **AC-RM** group should be considered as a subgroup with a relative accuracy of calculating staff performance forecasting indicators (**FTC-D** subgroup indicators) and staff performance evaluation and forecasting indicators (appropriate to the **MV-S** subgroup). Its accuracy depends on the accuracy of the methodology used in calculating the evaluation and forecasting indicator.

- The **AC-RC/RM** group has a relative parametric and methodological accuracy in calculating staff performance forecasting indicators, which include certain deterministic indicators of the **FTC-D** subgroup of forms and trends of changes of measured values:
  - the non-parametric weighted moving average smoothing method;
  - the non-parametric exponential smoothing method.

The **AC-RC/RM** group should be considered as a subgroup with a relative accuracy in calculating performance forecasting indicators, depending concurrently on the accuracy of the components and methodology used to calculate the forecast indicator.

It is clear that the only condition for maximizing the parametric accuracy of calculations of staff performance forecasting indicators (for the **AC-RC** and **AC-RC/RM** groups) is to maximize the accuracy in determining the corresponding evaluation and forecasting indicator components [the weighting coefficients

$\omega_{rij1\Delta T}$, $\omega_{rij2\Delta T}$, …, $\omega_{rij\tau_{max}\Delta T}$ (2), the weighting coefficient $\omega_{rij\tau\Delta T}$ (23), and the smoothing constant θ (24)].

Let us consider the conditions for maximizing the methodological accuracy of calculating staff productivity forecasting indicators (for the **AC-RM** and **AC-RC/RM** groups).

Based on the data shown in Table 3, the methodological accuracy of calculating forecast indicators will be determined by the following criteria (methods):

- using the least squares method for deterministic parameters that are calculated using linear, polynomial, exponential, power-law, and logarithmic approximations of the trend-line parametric functions for forecasting the parametric indicators appropriate to the **FTC-D** subgroup;

- using the method of estimating error and confidence intervals using Student's distribution for deterministic parameters calculated using non-parametric methods (simple moving average smoothing, weighted moving average smoothing, exponential approximation) for forecasting non-parametric indicators appropriate to the **FTC-D** subgroup;

- using the method of estimating the confidence interval with a given confidence level using the standard normal distribution function for forecasting the stochastic mathematical expectation **MV-S** subgroup indicator.

The next step is to analyze each of these criteria.

By the least squares method [16], the calculated deterministic forecasting parameter is accepted as the most accurate one under the condition of maximum proximity of the selected approximating function to the measured values ($\alpha_{rij1\Delta T\lambda}$, $\alpha_{rij2\Delta T\lambda}$, …, $\alpha_{rij\tau_{max}\Delta T\lambda}$), which should correspond to the minimum sum of squares of the difference between the corresponding approximating function and the measured values within the corresponding specific time intervals τ of measurement, i.e., when $\sum_{\tau=1}^{\tau_{max}}[F(\alpha_{rij\tau\Delta T\lambda}) - \alpha_{rij\tau\Delta T\lambda}]^2 \rightarrow \mathbf{min}$.

In turn, the maximum proximity of the selected approximating function to the measured values ($\alpha_{rij1\Delta T\lambda}$, $\alpha_{rij2\Delta T\lambda}$, …, $\alpha_{rij\tau_{max}\Delta T\lambda}$) should occur under condition of the highest (maximum) accuracy in determining the appropriate approximating function parameters. These include the constant coefficients $\mathbf{a}_{lin0}$ and $\mathbf{a}_{lin1}$ (12) of the linear approximation, the constant coefficients $\mathbf{a}_{pol0}$, $\mathbf{a}_{pol1}$, …, $\mathbf{a}_{poln}$ (13) of the polynomial trend, the constant coefficients $\mathbf{a}_{e0-1}$ and $\mathbf{a}_{e1-1}$ (14) or $\mathbf{a}_{e0-2}$ and $\mathbf{a}_{e1-2}$ (15) of the exponential approximation function, the constant coefficients $\mathbf{a}_{pow0}$ and $\mathbf{a}_{pow1}$ (18) of the power-law trend, and the constant coefficients $\mathbf{a_{ln0}}$ and $\mathbf{a_{ln1}}$ (20) of the logarithmic trend-line approximation. However, as follows from analysis of the least squares method, the accuracy in forecasting a particular approximating function does not directly depend on the number $\boldsymbol{\tau_{max}}$ (sample size) of the measured values.

By the method of estimating error and confidence intervals using Student's distribution [22], the deterministic forecasting parameter is taken to be the most accurate when the confidence interval $\Delta\alpha_{\mathbf{rij}\tau_{max+1}\Delta T\lambda}$ is minimized, i.e., when $\Delta\alpha_{\mathbf{rij}\tau_{max+1}\Delta T\lambda} = \mathbf{v}_{\tau,P} * \mathbf{s}_{\tau,\alpha} \rightarrow \mathbf{min}$ with the corresponding value $\nu_{\tau,\mathbf{P}}$ of Student's distribution with $\boldsymbol{\tau}$ ($\boldsymbol{\tau_{max}} - \mathbf{1}$) degrees of freedom and with the chosen confidence probability level $\mathbf{P}$ ($\mathbf{P}_{\Delta\alpha_{\mathbf{rij}\tau_{max+1}\Delta T\lambda}}$), and also with the reduced value $\mathbf{s}_{\tau,\alpha}$ ($\mathbf{s}_{\tau,\alpha} = \frac{\mathbf{S}_{\tau,\alpha}}{\sqrt{\boldsymbol{\tau}_{\mathbf{sample}}}}$) of the mean quadratic deviation $\mathbf{S}_{\tau,\alpha}$ ($\mathbf{S}_{\tau,\alpha} = \sqrt{\frac{\sum_{\tau=\tau_{max}-\tau_{sample}+1}^{\tau_{max}}(\alpha_{\mathbf{rij}\tau\Delta T\lambda} - \mathbf{z}_{\mathbf{rij}\tau_{max+1}\Delta T\lambda})^2}{\boldsymbol{\tau}_{\mathbf{sample}} - \mathbf{1}}}$) of each measured value ($\alpha_{\mathbf{rij}(\tau_{max}-\tau_{sample}+1)\Delta T\lambda}$, $\alpha_{\mathbf{rij}(\tau_{max}-\tau_{sample}+2)\Delta T\lambda}$, …, $\alpha_{\mathbf{rij}\tau_{max}\Delta T\lambda}$) from the sample $\boldsymbol{\tau}_{\mathbf{sample}}$ and the forecast value $\mathbf{z}_{\mathbf{rij}\tau_{max+1}\Delta T\lambda}$, i.e., either $\hat{\mathbf{z}}_{\mathbf{rij}\tau_{max+1}\Delta T\lambda}$ (22), or $\hat{\hat{\mathbf{z}}}_{\mathbf{rij}\tau_{max+1}\Delta T\lambda}$ (23), or $\tilde{\mathbf{z}}_{\mathbf{rij}\tau_{max+1}\Delta T\lambda}$ (24). The conditional criterion of $\Delta\alpha_{\mathbf{rij}\tau_{max+1}\Delta T\lambda} = \mathbf{v}_{\tau,P} * \mathbf{s}_{\tau,\alpha} \rightarrow \mathbf{min}$ is obviously achievable by minimizing the multipliers of the product $\nu_{\tau,\mathbf{P}} * \mathbf{s}_{\tau,\alpha}$, that is, provided that $\nu_{\tau,\mathbf{P}} \rightarrow \mathbf{min}$ and $\mathbf{s}_{\tau,\alpha} \rightarrow \mathbf{min}$.

In turn, as follows from analysis of the Student's distribution table [18], it is possible to achieve the condition $\nu_{\tau,\mathbf{P}} \rightarrow \mathbf{min}$ at a certain level of confidence probability $\mathbf{P}$ ($\mathbf{P}_{\Delta\alpha_{\mathbf{rij}\tau_{max+1}\Delta T\lambda}}$) by maximizing $\boldsymbol{\tau}_{\mathbf{sample}}$, that is, by maximizing the number (sample size) of measured values.

To reach concurrently the condition $\mathbf{s}_{\tau,\alpha} \rightarrow \mathbf{min}$, the procedure is as follows:

- $\boldsymbol{\tau}_{\mathbf{sample}}$ maximization, i.e., maximizing the number of measured values (the sample size);
- $\sum_{\tau=\tau_{max}-\tau_{sample}+1}^{\tau_{max}}(\alpha_{\mathbf{rij}\tau\Delta T\lambda} - \mathbf{z}_{\mathbf{rij}\tau_{max+1}\Delta T\lambda})^2$ minimization, that is, minimizing the absolute size of the difference between each of the measured values ($\alpha_{\mathbf{rij}(\tau_{max}-\tau_{sample}+1)\Delta T\lambda}$, $\alpha_{\mathbf{rij}(\tau_{max}-\tau_{sample}+2)\Delta T\lambda}$, …, $\alpha_{\mathbf{rij}\tau_{max}\Delta T\lambda}$) and the corresponding forecast value $\mathbf{z}_{\mathbf{rij}\tau_{max+1}\Delta T\lambda}$.

Based on the above, after certain mathematical operations are performed, it can be argued that to achieve the necessary measurement accuracy of the corresponding deterministic forecasting indicators, which corresponds to a level as close as possible to β (in a bilateral hypothesis) of the relative ratio between the confidence interval $\Delta\alpha_{\mathbf{rij}\tau_{max+1}\Delta T\lambda}$ and the forecasted performance indicator $\mathbf{z}_{\mathbf{rij}\tau_{max+1}\Delta T\lambda}$ at time $\boldsymbol{\tau}_{\mathbf{max+1}}$ (that is, when $\frac{\Delta\alpha_{\mathbf{rij}\tau_{max+1}\Delta T\lambda}}{\mathbf{z}_{\mathbf{rij}\tau_{max+1}\Delta T\lambda}} = \beta$) at a certain confidence probability level $\mathbf{P}_{\Delta\alpha_{\mathbf{rij}\tau_{max+1}\Delta T\lambda}} = \eta$ with the conditional assumption that each of the measured values ($\alpha_{rij(\tau_{max}-\tau_{sample}+1)\Delta T\lambda}$, $\alpha_{rij(\tau_{max}-\tau_{sample}+2)\Delta T\lambda}$, ..., $\alpha_{rij\tau_{max}\Delta T\lambda}$) will differ from the forecast value $z_{rij\tau_{max+1}\Delta T\lambda}$ in μ times, the minimum required sample size (the number τ_{sample}) of measured values can be determined as:

$$\tau_{sample} = \frac{\mu^2}{\beta^2} * \nu_{\tau,P_\eta}^{\ 2} + 1, \quad (34)$$

where:

ν_{τ,P_η} is the value of Student's distribution for τ ($\tau_{max} - 1$) degrees of freedom and for the selected confidence probability level P_η ($P_{\Delta\alpha_{rij\tau_{max+1}\Delta T\lambda}} = \eta$).

Table 4 shows example calculations of sample sizes τ_{sample} based on different values of conditional accuracy β ($\frac{\Delta\alpha_{rij\tau_{max+1}\Delta T\lambda}}{z_{rij\tau_{max+1}\Delta T\lambda}} * 100\% = 5\%$ and $\frac{\Delta\alpha_{rij\tau_{max+1}\Delta T\lambda}}{z_{rij\tau_{max+1}\Delta T\lambda}} * 100\% = 2{,}5\%$), different values of confidence probability levels η ($P_{\Delta\alpha_{rij\tau_{max+1}\Delta T\lambda}}$ = 90% and $P_{\Delta\alpha_{rij\tau_{max+1}\Delta T\lambda}}$ = 95%), and different conditional ratios μ of the measured values ($\alpha_{rij(\tau_{max}-\tau_{sample}+1)\Delta T\lambda}$, $\alpha_{rij(\tau_{max}-\tau_{sample}+2)\Delta T\lambda}$, ..., $\alpha_{rij\tau_{max}\Delta T\lambda}$) and forecasted values $z_{rij\tau_{max+1}\Delta T\lambda}$ (10%, 15%, and 20%).

Table 4

Examples of sample size (τ_{sample}) calculations at different conditional accuracies $\beta = \frac{\Delta\alpha_{rij\tau_{max+1}\Delta T\lambda}}{z_{rij\tau_{max+1}\Delta T\lambda}}$, different confidence probabilities η ($P_{\Delta\alpha_{rij\tau_{max+1}\Delta T\lambda}}$), and different conditional ratios μ of the measured values ($\alpha_{rij(\tau_{max}-\tau_{sample}+1)\Delta T\lambda}$, $\alpha_{rij(\tau_{max}-\tau_{sample}+2)\Delta T\lambda}$, ..., $\alpha_{rij\tau_{max}\Delta T\lambda}$) and the forecasted value $z_{rij\tau_{max+1}\Delta T\lambda}$.

#	$\frac{\Delta\alpha_{rij\tau_{max+1}\Delta T\lambda}}{z_{rij\tau_{max+1}\Delta T\lambda}} \sim$ β, %	$P_{\Delta\alpha_{rij\tau_{max+1}\Delta T\lambda}}$ η, %	Equal ratios $\alpha_{rij(\tau_{max}-\tau_{sample}+1)\Delta T\lambda}$, $\alpha_{rij(\tau_{max}-\tau_{sample}+2)\Delta T\lambda}$, ..., $\alpha_{rij\tau_{max}\Delta T\lambda}$ and $z_{rij\tau_{max+1}\Delta T\lambda}$ μ, %	$\tau_{sample} \geq$
1	5.0%	90%	10%	9
2	5.0%	90%	15%	17
3	5.0%	90%	20%	29
4	5.0%	95%	10%	14
5	5.0%	95%	15%	27
6	5.0%	95%	20%	46
7	2.5%	90%	10%	29
8	2.5%	90%	15%	61
9	2.5%	90%	20%	107
10	2.5%	95%	10%	46
11	2.5%	95%	15%	100
12	2.5%	95%	20%	177

By the method of estimating the confidence interval with a given confidence level using the standard normal distribution [23], the mathematical expectation as a stochastic forecasting parameter is taken as the most accurate while minimizing the confidence interval $z^{A_r/stoch3-4(ci)}_{\alpha^*-rij\Delta T\lambda}$, that is, when $z^{A_r/stoch3-4(ci)}_{\alpha^*-rij\Delta T\lambda} = \frac{z^{A_r/stoch2-1(sd)}_{rij\Delta T\lambda}}{\sqrt{\tau_{max}}} * p^{cl}_{\alpha-rij\Delta T\lambda} \rightarrow \min$ (32) with a corresponding confidence level $p^{cl}_{\alpha-rij\Delta T\lambda}$ that corresponds to the given confidence probability $p^{cp}_{\alpha-rij\Delta T\lambda}$ and with an appropriate standard deviation $z^{A_r/stoch2-1(sd)}_{rij\Delta T\lambda} = \sqrt{\sum_{\alpha^*=1}^{\alpha^*_{max}}[(\alpha_{\alpha^*-rij\Delta T\lambda} - z^{A_r/stoch1-1(me)}_{rij\Delta T\lambda})^2 * p_{\alpha^*-rij\Delta T\lambda}]}$ (26). Achieving a condition when $z^{A_r/stoch3-4(ci)}_{\alpha^*-rij\Delta T\lambda} \rightarrow \min$ is obviously possible only by maximizing the sample size (τ_{max}). Other components of the confidence interval $z^{A_r/stoch3-4(ci)}_{\alpha^*-rij\Delta T\lambda}$ calculation (32), $z^{A_r/stoch2-1(sd)}_{rij\Delta T\lambda}$ and $p^{cp}_{\alpha-rij\Delta T\lambda}$, which are determined either by the performance indicator values ($z^{A_r/stoch2-1(sd)}_{rij\Delta T\lambda}$) corresponding to the measurement results or by the confidence probability requirements ($p^{cp}_{\alpha-rij\Delta T\lambda}$), represent the initial data (or preconditions) to calculate the estimation accuracy given the measurements and therefore cannot be subject to changes to maximize accuracy.

Based on the above discussion, after the performance of simple mathematical operations, it can be argued that it is necessary to achieve the required accuracy of the stochastic forecasting parameter that corresponds to a level as close as possible to γ [this quantity represents the relative ratio of the confidence interval $z^{A_r/stoch3-4(ci)}_{\alpha^*-rij\Delta T\lambda}$ divided by the forecasted mathematical expectation $z^{A_r/stoch1-1(me)}_{rij\Delta T\lambda}$ (that is, when $\frac{z^{A_r/stoch3-4(ci)}_{\alpha^*-rij\Delta T\lambda}}{z^{A_r/stoch1-1(me)}_{rij\Delta T\lambda}} = \gamma$)] with a certain confidence level ψ (i.e., when $p^{cl}_{\alpha-rij\Delta T\lambda} = \psi$, which corresponds to the given confidence probability $p^{cp}_{\alpha-rij\Delta T\lambda} = \psi_P$) under the conditional assumption that the standard deviation $z^{A_r/stoch2-1(sd)}_{rij\Delta T\lambda}$ will differ from the forecasted mathematical expectation $z^{A_r/stoch1-1(me)}_{rij\Delta T\lambda}$ for χ times, the minimum required sample size (number τ_{max}) of the measured values can be calculated as:

$$\tau_{max} = \frac{\chi^2}{\gamma^2} * \psi^2. \quad (35)$$

Table 5 shows examples of calculating the sample size τ_{max} based on different conditional accuracies γ ($\frac{z^{A_r/stoch3-4(ci)}_{\alpha^*-rij\Delta T\lambda}}{z^{A_r/stoch1-1(me)}_{rij\Delta T\lambda}} * 100\% = 5\%$ and $\frac{z^{A_r/stoch3-4(ci)}_{\alpha^*-rij\Delta T\lambda}}{z^{A_r/stoch1-1(me)}_{rij\Delta T\lambda}} * 100\% = 2{,}5\%$), different confidence probabilities ψ_P ($p^{cp}_{\alpha-rij\Delta T\lambda} = 90\%$ and $p^{cp}_{\alpha-rij\Delta T\lambda} = 95\%$), and different conditional ratios χ of the standard deviation $z^{A_r/stoch2-1(sd)}_{rij\Delta T\lambda}$ and the mathematical expectation $z^{A_r/stoch1-1(me)}_{rij\Delta T\lambda}$ (10%, 15%, and 20%).

Table 5

Example calculations of the measured value sample sizes τ_{max} at different conditional accuracies $\gamma = \frac{z^{A_r/stoch3-4(ci)}_{\alpha^*-rij\Delta T\lambda}}{z^{A_r/stoch1-1(me)}_{rij\Delta T\lambda}}$, different confidence probabilities ψ_P ($p^{cp}_{\alpha-rij\Delta T\lambda}$), and different conditional ratios χ of the standard deviation $z^{A_r/stoch2-1(sd)}_{rij\Delta T\lambda}$ and the mathematical expectation $z^{A_r/stoch1-1(me)}_{rij\Delta T\lambda}$.

#	$\frac{z^{A_r/stoch3-4(ci)}_{\alpha^*-rij\Delta T\lambda}}{z^{A_r/stoch1-1(me)}_{rij\Delta T\lambda}} \sim$ γ, %	$p^{cp}_{\alpha-rij\Delta T\lambda}$ ψ_P, %	$p^{cl}_{\alpha-rij\Delta T\lambda}$ ψ, %	Equal ratios $z^{A_r/stoch2-1(sd)}_{rij\Delta T\lambda}$ and $z^{A_r/stoch1-1(me)}_{rij\Delta T\lambda}$ χ, %	$\tau_{max} \geq$
1	5.0%	90.1%	1.65	10%	11
2	5.0%	90.1%	1.65	15%	25
3	5.0%	90.1%	1.65	20%	43
4	5.0%	95.0%	1.96	10%	16
5	5.0%	95.0%	1.96	15%	35
6	5.0%	95.0%	1.96	20%	62
7	2.5%	90.1%	1.65	10%	44
8	2.5%	90.1%	1.65	15%	98
9	2.5%	90.1%	1.65	20%	175
10	2.5%	95.0%	1.96	10%	62
11	2.5%	95.0%	1.96	15%	139
12	2.5%	95.0%	1.96	20%	246

Based on the above cross-comparative analysis of deterministic and stochastic evaluation and forecasting of staff performance, the following general conclusions can be drawn regarding their practical feasibility:

1. When concurrently estimating the mean (**IC-mean**) and the dispersion (**IC-dispersion**) of the past performance of various employees working under the same conditions (**IC-e-condition**) in the same activity fields (**IC-e-direction**), the recommended indicators are the deterministic or stochastic characteristics given in Table 3 for the respective mean values of the **MV-D** subgroup or the mathematical expectation of the **MV-S** subgroup of mean indicators (measures of central tendency), as well as the mean absolute and/or relative variation and deviation ranges of the **VDR-D** subgroup or the standard deviation and variation coefficients of the **VDR-S** subgroup of the measured values of variation and deviation ranges (measures of spread).

2. When concurrently estimating the mean (**IC-mean**), dispersion (**IC-dispersion**) and trends of changes (**IC-change**) of the future performance of various employees working under the same conditions (**IC-e-condition**) in the same areas of activity (**IC-e-direction**), the recommended indicators are the corresponding mathematical expectation stochastic characteristics given in Table 3 for the **MV-S** subgroup of mean values (measures of central tendency), the standard deviations and coefficients of variation of the **VDR-S** subgroup of variation and deviation ranges (measured of spread), and the skewness and kurtosis coefficients, the actual and reduced probabilities, and the absolute and reduced confidence intervals of the **FTC-S** subgroup of the forms or trends of changes in the measured values (probability measures).

3. When assessing the past mean (**IC-mean**) and dispersion (**IC-dispersion**) and at the same time the mean (**IC-mean**), dispersion (**IC-dispersion**) and trends of changes (**IC-change**) of future performance for various employees working under the same conditions (**IC-e-condition**) and in the same areas of activity (**IC-e-direction**), the recommended indicators are the corresponding mathematical expectation stochastic characteristics given in Table 3 for the **MV-S** subgroup of mean values (measures of central tendency), the standard deviations and coefficients of variation of the **VDR-S** subgroup of variation and deviation ranges (measures of spread), and the skewness and kurtosis coefficients, the actual and reduced probabilities, and the absolute and reduced confidence intervals of the **FTC-S** subgroup of the forms or trends of changes in the measured values (probability measures).

4. When assessing the mean (**IC-mean**) and dispersion (**IC-dispersion**) in the past and at the same time the mean (**IC-mean**), dispersion (**IC-dispersion**), and trends of changes (**IC-change**) of performance in the future for various employees working under different conditions (**IC-d-condition**) in different areas of activity (**IC-d-direction**), the recommended procedure is to carry out for further use the mathematical transformation (reduction, scaling, etc.) of all the deterministic and stochastic characteristics given in Table 3 for the **MV-D** and **MV-S** subgroups of means (measures of central tendency), the **VDR-D** and **VDR-S** subgroups of variation and deviation ranges (measures of spread), and the **FTC-D** and **FTC-S** subgroups of forms and trends of changes (probability measures) of the measured values. This mathematical scaling can be carried out by converting the entire initial sequence of all absolute measured performance indicators of all employees of the **j**-th enterprise in the measurement time interval $\Delta \mathbf{T}$ into the corresponding reduced sequence of all measured relative performance indicators of all employees of the **j**-th enterprise in the

given measurement time interval ΔT by grouping (uniting into groups) all measured absolute indicators into the same number **G** of groups for all performance indicators for all estimated employees of the **j**-th enterprise. The number of groups **G** is determined by each enterprise separately. For example, according to [22], the number of such groups can be as large as 10. This sequence of all relative measured performance indicators reduced to **G** groups, as well as the corresponding initial sequence of absolute measured performance indicators, shown in Table 1, can also be summarized for clarity in the appropriate registration form for evaluating and forecasting relative measurement results, e.g., in the form shown below in Table 6 and/or Figure 3.

Table 6
Register of the reduced measured values of the performance parameter **r** for the **i**-th employee of the **j**-th enterprise at measurement time interval ΔT.

Parameter	Fixed time τ of measurement carried out with measurement step Δt				
	1	**2**	**3**	**...**	τ_{max}
Value of indicator **r**	$\overline{\overline{\alpha_{1\lambda}}}$	$\overline{\overline{\alpha_{2\lambda}}}$	$\overline{\overline{\alpha_{3\lambda}}}$	...	$\overline{\overline{\alpha_{\tau_{max}\lambda}}}$

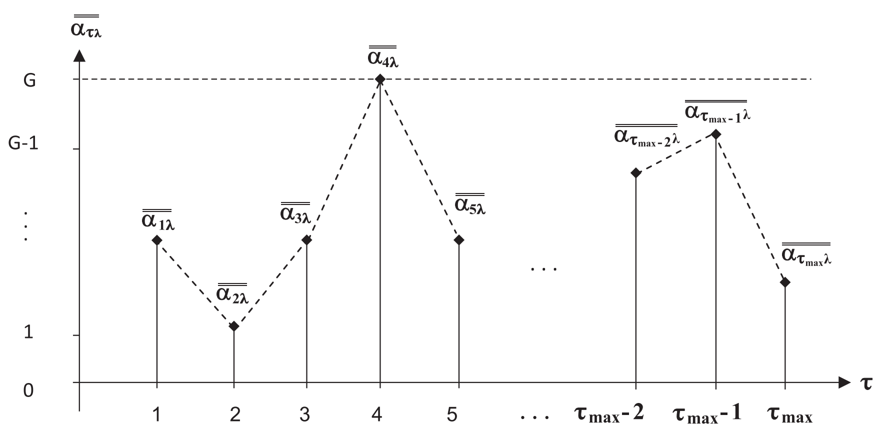

Figure 3
Reduced measured values of performance parameter **r** for the **i**-th employee of the **j**-th enterprise at measurement time interval ΔT.

Using the reduced sequence for all the measured relative performance indicators of all the employees of the **j**-th enterprise along the measurement time interval ΔT provides a basis for calculating all scalable (reduced) deterministic and stochastic characteristics of the mean indicator subgroups **MV-D** and **MV-S** (measures of central tendency), the subgroups of variation and deviation ranges **VDR-D** and **VDR-S** (measures of spread), and the subgroups of the forms or trends of changes **FTC-D** and **FTC-S** (probability measures) of measured values, the procedure being similar to the non-scalable characteristics calculation given in Table 3.

In accordance with the above general conclusions on the feasibility of practical use of deterministic and stochastic staff performance evaluation and forecasting (in clauses 1-3), relevant scalable features can be used to estimate the mean level (**IC-mean**) and the level of dispersion (**IC-dispersion**) in the past and at the same time, as well as the mean level (**IC-mean**), the level of dispersion (**IC-dispersion**), and the level of change trends (**IC-change**) in performance in the future for different employees working under different conditions (**IC-d-condition**) and in different activities (**IC-d-direction**).

Such scaling also opens up prospects for performance evaluation and forecasting, not only of individual employees, but also of work teams carrying out their tasks under different conditions (**IC-d-condition**) and in different areas of activity (**IC-d-direction**).

5. To achieve acceptable accuracy (usually the conditional accuracy, $\frac{z_{\alpha^*-rij\Delta T\lambda}^{A_r/stoch3-4(ci)}}{z_{rij\Delta T\lambda}^{A_r/stoch1-1(me)}} * 100\% \leq 5\%$, a confidence probability level $p_{\alpha-rij\Delta T\lambda}^{cp} \leq 95\%$, and a conditional ratio between standard deviation and mathematical expectation of $\frac{z_{rij\Delta T\lambda}^{A_r/stoch2-1(sd)}}{z_{rij\Delta T\lambda}^{A_r/stoch1-1(me)}} * 100\% \leq 10\%$) for estimating the mathematical expectation in the stochastic subgroup of mean indicators **MV-S** (measures of central tendency) in the past and simultaneously in the future for various employees working under the same or different conditions (**IC-e-condition** and **IC-d-condition**) in the same or different areas of activity (**IC-e-direction** and **IC-d-direction**), a measured value sample size $\tau_{max} \geq 16$ is needed (Table 5).

Based on the above, it is possible to achieve the specified minimum required accuracy in 1 year and 4 months (with staff performance indicators measured once a month), or in approximately 4 months (when staff performance indicators are measured once a week), or in about 3 weeks (when staff performance indicators are measured once a day). This analysis clearly illustrates the need for daily staff performance measurement. Note that the importance of daily performance measurement is also indicated, for example, in [12].

In addition to achieving the desired stochastic evaluation and forecasting accuracy for measured values, it is extremely important to strive for one more quality characteristic in stochastic evaluation and forecasting: to reduce the sample volatility of the measured values [27, 28, 29], which in the context of average annual measurement can be defined as

$$\sigma_Y = \frac{\mathbf{z}_{\mathbf{rijY\lambda}}^{\mathbf{A_r/stoch2-1(sd)}}}{\sqrt{\frac{\mathbf{N_C}}{\mathbf{N_Y}}}}, \tag{36}$$

where:

$\mathbf{z}_{\mathbf{rijY\lambda}}^{\mathbf{A_r/stoch2-1(sd)}}$ is the average annual standard deviation of the sequence of measured values;

$\mathbf{N_C}$ is the number of days in a year when staff performance indicators were measured;

$\mathbf{N_Y}$ is the number of working days per year.

Based on analysis of Eq. (36), the greater the number of days in the year when staff performance indicators were measured, the lesser will be the average annual volatility of the sample of measured values. This implies that with approximately 250 working days per year and the uniformity average annual standard deviation $\mathbf{z}_{\mathbf{rijY\lambda}}^{\mathbf{A_r/stoch2-1(sd)}}$, the volatility of a sample of measured values of staff performance indicators measured daily will be approximately 4.8 times less than for a single weekly measurement and approximately 20.8 times less than for a single monthly measurement.

Chapter 5

Results of Practical Application of the Stochastic Approach to Staff Performance Evaluation and Forecasting

The possibility of practical use of a stochastic indicator combined set (**SICS**) was tested for staff evaluation and forecasting at four companies in different market segments [insurance, FMCG (fast-moving consumer goods), telecommunications, and recruitment] with different performance indicators, with different sizes of staff performance indicator samples, and with different performance indicator measurement frequencies.

Without involving the names and specific segment specializations of the companies or the specific staff performance indicators selected for evaluation and forecasting by these companies, the results of using **SICS** for different sample sizes under different frequencies of staff performance indicator measurement for each of the four companies were analyzed.

Based on the analysis results, the following findings were established:

– For company #1 (insurance company): with a sample size of 36 indicators and a performance indicator measurement frequency of once per calendar month (that is, with a sample of 36 monthly indicators for each staff performance indicator for each month for three years), all the **SICS** indicators were assessed for each of the 11 staff members in the sample.

The results of **SICS** staff performance indicator analysis for company #1 showed moderately low stability of the measured staff performance indicators. The coefficients of variation showed, instead of acceptable (permissible) stability values of 10%–15%, a range from 45.64% to 591.61%, with an average coefficient of variation of 192.29% for all 11 estimated employees. Among the 11 measured values of the coefficient of variation, no indicator (no employee) had levels within the acceptable limits of stability (10%–15%).

Moreover, the measured staff performance indicator evaluation and forecasting results at company #1 showed rather low accuracy, partly because of the already mentioned low stability. The values of the relative confidence intervals, instead of showing acceptable (permitted) accuracy values of 5%–10% (with a confidence level of 95%), ranged from 14.91% to 193.26%, with an average confidence interval of 62.81% for all 11 estimated employees. Among the 11 measured values of confidence intervals (for each of the 11 employees), none of the indicators had levels within the acceptable accuracy limits (5%–10%).

The achievement of a minimally acceptable (10%) accuracy level (for example, the average confidence interval) in staff performance evaluation and forecasting at company #1 would be practically possible (for a sample size of 36 indicators) with an increase in average staff activity stability (coefficient of variation) to ~30.6%, that is, ~ 6.284 times, or with the current stability factor, by increasing the sample size to ~1420 measurements, i.e., ~ 39.444 times the current sample size (measuring staff performance indicators once per month), measurement of the required staff performance indicators would take more than 118 years, which is practically impossible.

– For company #2 (the FMCG company), with a sample of 36 indicators and a frequency of measurement of once per calendar month (that is, in a sample of 36 monthly indicators with one staff performance indicator registered each month for three years), all the **SICS** indicators were assessed for each of the 38 staff members in the sample.

The results of **SICS** staff performance indicator analysis for company #2 showed a relatively low stability of measured staff performance. The coefficients of variation, instead of showing acceptable (permissible) stability values of 10%–15%, ranged from 12.34% to 412.31%, with an average coefficient of variation of 88.70% for all 38 estimated staff members. Among the 38 measured coefficients of variation (for each of the 38 employees), only 1 value (1 employee) was observed at a level within the acceptable stability limits (10%–15%), at 12.34%.

Moreover, for company #2, the measured staff performance indicators also showed rather low assessment accuracy, partly because of the already mentioned low stability. The relative confidence intervals, instead of showing acceptable (permissible) accuracy values of 5%–10% (with a confidence level of 95%), ranged from 4.03% to 134.69%, with an average confidence interval of 28.98% for all 38 estimated staff members. Among the 38 measured confidence intervals, only 6 registered values (6 estimated employees) had levels within the acceptable accuracy limits (5%–10%), with values of 4.03%, 5.05%, 7.70%, 8.08%, 8.99%, and 9.65%.

The achievement of a minimally acceptable (10%) level of accuracy (in example, the average confidence interval) in staff performance evaluation and forecasting in the company #2 would be difficult. It could be done either (while maintaining the sample size of 36 indicators) by increasing the average stability of staff activity measurement (the coefficient of variation) to ~30.6%, i.e., ~2.899 times, or (at the current stability level) by increasing the sample size to ~302 measurements, i.e., 8.39 times, that used in this study (maintaining the applied performance indicator measurement frequency as once per month). This would require measuring staff performance indicators over a period of 25 years or more.

– For company #3 (a telecommunications company), samples of 55 to 80 performance indicators were measured once per day at the end of the working day (that is, for a sample of 55 to 80 daily indicators per staff performance indicator per day during four calendar months with approximately 20 working days in each month). All **SICS** indicators were assessed for each of the 12 staff members in the sample.

The results of the **SICS** staff indicator analysis at company #3 identified a relatively acceptable overall stability of estimated staff performance. The coefficients of variation, compared to acceptable (permissible) stability values of 10%–15%, ranged from 8.65% to 24.46%, with an average coefficient of variation of 14.51% for all 12 estimated staff members. Among the 12 measured coefficients of variation (for each of the 12 employees), only 4 values (4 employees) had levels beyond the acceptable stability limits (10%–15%), at 17.09%, 21.92%, 22.19%, and 24.46%.

Moreover, the measured staff performance indicator assessment among company #3 staff showed moderately high accuracy, partly due to the already indicated relatively acceptable stability parameter. The relative confidence intervals, compared to acceptable (permissible) accuracy values of 5%–10% (with a confidence level of 95%), ranged from 1.91% to 5.36%, with an average confidence interval of 3.36% for all 12 estimated employees.

– For company #4 (a recruitment company), the registered samples numbered 93, 219, and 228 indicators, with the staff performance indicators measured once daily at the end of each working day. In other words, samples of 93, 219, and 228 daily indicators for each staff performance indicator per day were taken for approximately 5, 11, and 12 calendar months with approximately 20 working days in each month. All **SICS** indicators for each of the 3 estimated staff members were assessed.

The results of staff **SICS** indicator analysis in company #4 identified a low stability of measured performance indicators for each of the 3 estimated staff members. The registered coefficients of variation, compared to acceptable (permissible) stability values of 10%–15%, were 32.56%, 42.35%, and 46.93%, with an average coefficient of variation of 40.61% for all three estimated employees.

However, the accuracy of measured performance indicator evaluation and forecasting for each of the three estimated employees was relatively acceptable. The relative confidence intervals, compared to acceptable (permissible) accuracy values of 5%–10% (at a confidence level of 95%), were 4.72%, 5.31%, and 5.57%, with an average confidence interval of 5.14% for all 3 estimated employees.

Conclusions

This book provides a detailed comparative analysis of the use of deterministic and stochastic approaches to staff performance evaluation and forecasting.

The analysis presented above formalizes important definitions and concepts, including the definition and concept of staff performance evaluation and forecasting, the definition and concept of employee efficiency, and that of identifying the essence of deterministic and stochastic approaches and characteristics to evaluate and forecast staff effectiveness. This book presents a complex of stability concepts, the prospects for improving or worsening staff performance, the prospects for changing the speed of improvement or deterioration, the probability of achieving certain staff performance values, and the accuracy of evaluating and forecasting the corresponding staff performance parameters for past and future periods. In addition, the discussion addresses the concept of indicators for cross-comparative analysis in deterministic and stochastic staff performance evaluation and forecasting, an understanding of parametric and methodological accuracy in calculating staff performance evaluation and forecasting indicators, determining the minimum required sample size of measured values to achieve the desired accuracy of staff performance evaluation and forecasting, and illustrating the advantages of daily measurement of staff performance, including improving the level of staff performance evaluation and forecasting accuracy and reducing the sample average annual volatility of measured staff performance values.

To formalize the results of this staff performance evaluation and forecasting study, it is proposed to apply the sequence $\mathbf{A_r}$ of parameters $\alpha_{\mathbf{rij\tau\Delta T\lambda}}$ that should characterize a quantitative value α of the corresponding measured **r**-th performance indicator of the corresponding **i**-th employee of the corresponding **j**-th enterprise at the corresponding defined time $\boldsymbol{\tau}$ for the corresponding defined measurement interval $\Delta\mathbf{T}$ in the corresponding defined unit of measurement $\boldsymbol{\lambda}$.

It is also proposed to consider the activity efficiency of the **i**-th employee of the **j**-th enterprise according to the measured performance indicator **r** in the measurement time interval $\Delta\mathbf{T}$ as the degree to which the characteristics set $\mathbf{Z_{A_r}}\,(\mathbf{z^{A_r-1}_{rij\Delta T\lambda}}, \mathbf{z^{A_r-2}_{rij\Delta T\lambda}}, \ldots, \mathbf{z^{A_r-Z_{A_r}}_{rij\Delta T\lambda}})$ of the initial sequence $\mathbf{A_r}$ of measured values $\alpha_{\mathbf{rij1\Delta T\lambda}}$, $\alpha_{\mathbf{rij2\Delta T\lambda}}$,

..., $\alpha_{\mathbf{rij}\tau_{max}\Delta\mathbf{T}\lambda}$ achieves the corresponding expected levels $\mathbf{z}^{\mathbf{eff-1}}_{\mathbf{rij}\Delta\mathbf{T}\lambda}$, $\mathbf{z}^{\mathbf{eff-2}}_{\mathbf{rij}\Delta\mathbf{T}\lambda}$, ..., $\mathbf{z}^{\mathbf{eff-Z_{max-eff}}}_{\mathbf{rij}\Delta\mathbf{T}\lambda}$ of the reference value set $\mathbf{Z_{eff}}$.

In this case, direct calculation of the characteristics set $\mathbf{Z}_{\mathbf{A_r}}$ of the specified initial sequence $\mathbf{A_r}$ can potentially be carried out using two main approaches: the deterministic approach and its opposite, the stochastic approach.

It is suggested that such a process be considered as a deterministic approach when relevant calculations or research are carried out to provide an accurate evaluation or a forecast with certainty of the previously discussed quantitative mean indicators in the **MV-D** subgroup, the quantitative indicators of variation and deviation ranges in the **VDR-D** subgroup, and the quantitative indicators of forms or trends of changes in the **FTC-D** subgroup to characterize staff performance.

The process should rather be considered as a stochastic approach when the calculations are based on a random probability distribution of the investigated value or sequence that can be analyzed statistically, but cannot be forecast accurately. In other words, a stochastic approach is characterized by a hypothesis and uses in its calculations random values of the considered probabilistic mean indicators from the **MV-S** subgroup, the probabilistic indicators of variation and deviation of ranges from the **VDR-S** subgroup, and the probabilistic indicators of forms or trends of changes from the **FTC-S** subgroup concerning staff performance.

It is proposed to treat in practice the quantitative and probabilistic mean indicators of the **MV-D** and **MV-S** subgroups as absolute staff performance indicators for the past and the past-and-future periods respectively, and to treat quantitative and probabilistic indicators of variation and deviation ranges of the **VDR-D** and **VDR-S** subgroups as staff performance stability indicators for the past and past-and-future periods respectively. At the same time, quantitative indicators of forms or trends of changes (**FTC-D** subgroup) are proposed to be considered in practice as absolute forecasting indicators of staff performance in future periods. Apart from that, the probabilistic indicators of forms or trends of changes (**FTC-S** subgroup) are proposed to be considered in practice as: Karl Pearson’s coefficient of skewness, to provide an indicator of improving or worsening prospects to represent relevant staff performance for past and future periods, and Karl Pearson’s coefficient of kurtosis, to provide an indicator of the prospective speed of improvement or worsening of relevant staff performance for past and future periods. Additionally, the probability shall be considered here as the staff's potential to achieve certain performance values in future periods, and the confidence interval of assessment shall be applied to express the accuracy of the estimated mathematical expectation of the staff performance parameter for past and future periods.

A cross-comparative analysis of deterministic and stochastic staff performance assessment is proposed using five main cross-comparison indicators: the **PF** (past-future) indicator of the possibility of using indicators to estimate staff performance in past and/or future periods, and the indicators **IC** (information content), **AC** (accuracy), **SM** (simplicity), and **CS** (cost).

At this point, it is established that all deterministic and stochastic indicators for evaluating and forecasting staff performance are in a certain sense almost identical considering the simplicity **SM** and cost **CS** parameters.

It has been established that according to the **PF** indicator, all deterministic indicators in the **MV-D** subgroup of mean indicators and the **VDR-D** subgroup of measured value variation and deviation ranges can be used to evaluate staff performance exclusively in the past. In turn, for the specified **PF** indicator, all deterministic indicators appropriate to the **FTC-D** subgroup of the forms or trends of changes in measured values can be used to forecast the degree of staff performance exclusively in the future. All stochastic indicators belonging to the **MV-S** subgroup of means, the **VDR-S** subgroup of variation and deviation ranges, and the **FTC-S** subgroup of the forms or trends of changes in measured values may be used to estimate past and future performance. The possibility of applying the stochastic indicators from the **MV-S**, **VDR-S**, and **FTC-S** subgroups to simultaneously evaluate and forecast past and future staff performance clearly demonstrates their appropriate flexibility and thus the advantage of using stochastic subgroups rather than the corresponding deterministic subgroups **MV-D**, **VDR-D**, and **FTC-D** of staff performance evaluation and forecasting indicators.

As for the information level characteristic (according to the **IC** indicator), the set of deterministic indicators that is appropriate to the **MV-D**, **VDR-D**, and **FTC-D** subgroups and can be used to evaluate previous (past) and forecasting further (future) absolute staff performance parameters and to evaluate past performance stability for staff working under the same (**IC-e-condition**) and different (**IC-d-condition**) conditions and in the same (**IC-e-direction**) and different (**IC-d-direction**) activity fields. Furthermore, according to the **IC** indicator, the stochastic indicator sets in the **MV-S**, **VDR-S**, and **FTC-S** subgroups can be used to evaluate previous (past) and to forecast further (future) absolute staff performance indicators, to evaluate past and to forecast future indicators of staff performance stability, and to evaluate past and to forecast future indicators of the prospects of staff performance improving or worsening, indicators of the prospective speed of staff performance improving or worsening, indicators of the probability that evaluated and forecasted staff performance achieves certain performance values, and indicators of accuracy in evaluating and forecasting the mathematical expectation of staff performance, all this for staff working under the same (**IC-e-condition**) and different (**IC-d-condition**) conditions and in the same (**IC-e-direction**) and different (**IC-d-direction**) areas of activity. Use of the stochastic indicators in the **MV-S**, **VDR-S**, and **FTC-S** subgroups to evaluate and forecast a wide variety of staff performance indicators (indicators of prospects, of the rate of change in prospects, of probability, and of accuracy) also clearly demonstrates their appropriate universality, as well as a certain advantage in using stochastic subgroups rather than the corresponding deterministic subgroups **MV-D**, **VDR-D**, and **FTC-D** of indicators for evaluating and forecasting staff performance.

It should also be emphasized that when the entire initial sequence of all measured absolute performance indicators for all employees of an enterprise in a certain measurable time interval in the corresponding reduced sequence and grouped (united by groups) by all measured absolute indicators is possibly transformed into the same number of groups of deterministic and stochastic indicators in the **MV-D**, **VDR-D**, **FTC-D**, **MV-S**, **VDR-S**, and **FTC-S** subgroups for all the estimated enterprise's employees, the reduced measured performance indicators can be used to evaluate and forecast the performance of both individual employees and employee teams working under the same (**IC-e-condition**) or different (**IC-d-condition**) conditions and in the same (**IC-e-direction**) or different (**IC-d-direction**) directions of staff activity.

In addition, it has been determined that according to the **AC** accuracy parameter, all deterministic indicators (except for weighted indicators) in the **MV-D** and **VDR-D** subgroups and all stochastic indicators in the **VDR-S** and **FTC-S** subgroups should be considered as indicators of absolute (i.e., maximum) accuracy. In turn, all deterministic weighted indicators in the **MV-D** subgroups should be considered as relative accuracy indicators that depend on the accuracy of the parametric components in the estimated indicator calculation. All deterministic indicators (except for indicators obtained by the weighted moving average and exponential approximation methods) among the **FTC-D** and **MV-S** subgroups' stochastic indicators should also be considered as indicators of relative accuracy that depend on the methodological accuracy of the evaluation and forecasting indicator calculations. In addition, deterministic indicators obtained by the weighted moving average method and the exponential approximation method that belong to the **FTC-D** subgroup should also be considered as indicators of relative accuracy, depending on both the parametric accuracy of calculating the forecasting indicator components and the methodological accuracy of the forecasting indicator calculations. Additionally, assuming that the mathematical expectation $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/stoch1-1(me)}}$ (an indicator in the **MV-S** subgroup) can be considered as the mean value $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/det1-1(am)}}$ (an indicator in the **MV-D** subgroup), i.e., given that $\mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/det1-1(am)}} = \mathbf{z}_{\mathbf{rij\Delta T\lambda}}^{\mathbf{A_r/stoch1-1(me)}}$, it should be noted that the accuracy in calculating the **MV-S** subgroup's stochastic indicator can also be considered as an absolute indicator (i.e., the maximum). The same holds for the accuracy of all other stochastic indicators in the **VDR-S** and **FTC-S** subgroups.

Based on the foregoing, and considering the previously defined universality of stochastic indicators for the **PF** parameter and their universality for the **IC** parameter, the use of stochastic indicators belonging to the **MV-S**, **VDR-S**, and **FTC-S** subgroups is comprehensively (including all five cross-comparison indicators) superior to the use of the indicators in the corresponding deterministic subgroups **MV-D**, **VDR-D**, and **FTC-D** for evaluating and forecasting staff performance.

In addition, it is recommended to use the least squares method, the methods of estimating error and confidence intervals using the Student's distribution, and the estimation of confidence intervals with a given confidence level using the standard normal distribution to determine the degree to which the necessary methodological accuracy has been achieved in calculating the deterministic indicators of the **FTC-D** subgroup and the stochastic indicators of the **MV-S** subgroup.

In this case, the corresponding Equation (34) determined the minimum required sample size τ_{sample} of measured values that must be provided to achieve the necessary measurement accuracy of the corresponding deterministic forecasts for the **FTC-D**, subgroup. Specifically, $\tau_{sample} = \frac{\mu^2}{\beta^2} * \nu_{\tau,P_\eta}^2 + 1$, with the given values of μ (the ratio of a performance indicator's measured values to its forecasted values), β (the ratio of the confidence interval to the forecasted performance indicator), η (a certain level of confidence probability), and ν_{τ,P_η} (the value of Student's distribution with τ degrees of freedom at the desired confidence probability level η).

In addition, Equation (35) determines the minimum required sample size τ_{max} of measured values that must be provided to achieve the necessary measurement accuracy for the corresponding stochastic forecasting indicator in subgroup **MV-S**, namely $\tau_{max} = \frac{\chi^2}{\gamma^2} * \psi^2$ at the given values of χ (the ratio of the standard deviation to the forecasted mathematical expectation of the performance indicator), γ (the ratio of the confidence interval to the forecasted mathematical expectation of the performance indicator), and ψ (the defined confidence level).

It was also established that, in its turn, the estimated measurement accuracy of both deterministic and stochastic indicators depends on the sample size, the stability of the measured values, and the confidence level, but does not directly depend on the interval (on a daily, weekly, or monthly basis) assigned to obtain the measured values. Based on this, with a clear desire to obtain evaluations and forecasts of staff performance in a shorter time, the use of daily staff performance measurement has an undeniable advantage over measurements on a weekly or monthly basis.

The advantage of daily measurements of staff performance becomes even more important with the need to reduce the average annual volatility (36) of the sample of measured values of staff productivity. This can be achieved by increasing the number of staff performance measurements per year. The volatility of the sample of measured values of staff performance indicators with daily measurement will be approximately 4.8 times less than with a single weekly measurement and approximately 20.8 times less than with a single monthly measurement.

Assessing the results of the comparative analysis of deterministic and stochastic staff performance evaluation and forecasting on the relevant indicators **PF**, **IC**, **AC**, **SM**, and **CS**, as well as considering the requirements for achieving the needed staff performance estimation accuracy and the measured performance values' sample volatility, it is possible to speak of the predominant practical feasibility of daily measurement of staff performance using the combined set of stochastic indicators (**SICS**) for all three subgroups: **MV-S**, the mean value (mathematical expectation) subgroup; **VDR-S**, the variation and deviation ranges (standard deviation and coefficient of variation) subgroup; and the **FTC-S** subgroup of forms and trends of changes (coefficients of skewness and kurtosis, probability of achieving certain performance levels, and confidence interval). This statement in favor of stochastic (rather than deterministic) indicators is simply logical and natural, provided that the results of staff activities are stochastic in nature (but not deterministic) and depend on the combination of many inherently random (probabilistic) external and internal factors.

The combined set of stochastic performance indicators (**SICS**) for an individual employee, a group of employees, or the entire company should in some sense be considered as a "professional portrait" of the corresponding staff member, team, or company.

Clearly, practical daily use of the **SICS** combined set of stochastic indicators should ensure simultaneous compliance with the requirements of stochastic assessment accuracy and the requirements concerning the volatility of the sample of measured staff performance values.

These conclusions have been confirmed by practical application of **SICS** as a combined set of stochastic indicators for staff evaluation and forecasting at four companies operating in different market segments (insurance, FMCG, telecommunications, and recruiting) with different performance indicators, different staff performance sample sizes, and different frequencies of performance indicator measurement.

The results obtained by deterministic and stochastic staff performance assessment can be applied in practice by the appropriate HR managers to evaluate and forecast the performance of a wide range of professionals in various fields in a wide range of industries.

These approaches are easy to implement and do not require significant basic or special training for application.

In addition, use of the analyzed approaches will make it possible to obtain a prompt (with daily measurement) assessment of the need, subject matter, and frequency of staff training according to certain parameters to achieve the necessary assessment levels, performance stability, prospects, probability of occurrence, and accuracy, as well as to estimate promptly the effectiveness of training, which in turn will effectively and promptly improve the professional level of the company's staff.

References

1. Nicholas Burkholder, Scott Golas, Jeremy Shapiro. Ultimate Performance: Measuring Human Resources at Work. 2007. ISBN: 9780471741213. 288 pages.
2. Ozlem Olgu, Hasan Dincer, Umit Hacioglu. Handbook of Research on Strategic Developments and Regulatory Practice in Global Finance. 2015. ISBN: 9781466672895. 433 pages.
3. Andrew Neely, Chris Adams, Mike Kennerly. The Performance Prism: The Scorecard for Measuring and Managing Business Success. 2002. ISBN: 0273653342. 393 pages.
4. Jac Fitz-enz. The ROI of Human Capital: Measuring the Economic Value of Employee Performance. 2009. ISBN: 9780814413326. 312 pages.
5. David Parmenter. Key Performance Indicators: Developing, Implementing, and Using Winning KPIs. 2019. ISBN: 9781119620778. 384 pages.
6. Bernard Marr. Key Performance Indicators: The 75 Measures Every Manager Needs to Know. 2012. ISBN: 9780273750116. 347 pages.
7. Student's Oxford Canadian Dictionary. Second edition. 2007. ISBN: 9780195427158. 1286 pages.
8. David Scott. The American Heritage Dictionary of Business Terms. 2009. ISBN: 9780618755257. 594 pages.
9. Oxford Advanced Learner's Dictionary of Current English. Ninth edition. 2015. ISBN: 9780194798860. 1820 pages.
10. Christopher Clapham, James Nicholson. The Concise Oxford Dictionary of Mathematics. Fifth edition. 2014. ISBN: 9780199679591. 537 pages.
11. Graham Upton, Ian Cook. Oxford Dictionary of Statistics. Third edition. 2014. ISBN: 9780199679188. 488 pages.
12. Stewart Liff. Managing Government Employees: How to Motivate Your People, Deal with Difficult Issues, and Achieve Tangible Results. 2007. ISBN: 9780814408872. 223 pages.
13. The American Heritage Dictionary of the English Language. Fifth edition. 2018. ISBN: 9781328841698. 2084 pages.

14. David Promislow. Fundamentals of Actuarial Mathematics. 2015. ISBN: 9781118782460. 552 pages.

15. Yadolah Dodge. The Oxford Dictionary of Statistical Terms. Sixth edition. 2003. ISBN: 0198509944. 498 pages.

16. Gerard Verschuuren. Excel 2013 for Scientists. Revised & expanded third edition. 2014. ISBN: 9781615470259. 321 pages.

17. Jeffrey Camm. Essentials of Business Analytics. Second edition. 2017. ISBN: 9781305627734. 870 pages.

18. Murray Spiegel, Seymour Lipschutz, John Liu. Schaum's Outlines: Mathematical Handbook of Formulas and Tables. Fifth edition. 2017. ISBN: 9781260010534. 309 pages.

19. Dennis Zill. Advanced Engineering Mathematics. Sixth edition. 2018. ISBN: 9781284105902. 1024 pages.

20. Charles Kirkpatrick, Julie Dahlquist. Technical Analysis: The Complete Resource for Financial Market Technicians. Third edition. 2015. ISBN: 9780134137186. 704 pages.

21. Charles Kirkpatrick. Study Guide for the Second Edition of Technical Analysis: The Complete Resource for Financial Market Technicians. 2013. ISBN: 9780133092608. 272 pages.

22. Vladimir Efimov. Statistical Methods in Quality Management. 2003. ISBN: 5891463500. 138 pages.

23. J. K. Sharma. Business Statistics. Second edition. 2006. ISBN: 9788177586541. 750 pages.

24. Sheldon Ross. Introduction to Probability Models. Eleventh edition. 2014. ISBN: 9780124079489. 767 pages.

25. Robert Donnelly. Fatma Abdel-Raouf. Statistics. Third edition. 2016. ISBN: 9781465451668. 336 pages.

26. Prabhanjan Narayanachar Tattar. Statistical Application Development with R and Python: Power of Statistics Using R and Python. Second edition. 2017. ISBN: 9781788622264. 432 pages.

27. Sheldon Natenberg. Option Volatility and Pricing: Advanced Trading Strategies and Techniques. Second edition. 2014. ISBN: 9780071818773. 512 pages.

28. Steven Place. Timing Volatility: Measure Fear and Greed to Get an Edge in the Market. 2011. ISBN: 9780132942911. 30 pages.

29. Adam Iqbal. Volatility: Practical Options Theory. 2018. ISBN: 9781119501688. 208 pages.

lat&k
publisher

Editor: Diana Bouchard

The translation from Ukrainian to English
by "Exleng" Translation Agency

Technical editor: Victor Bykhun

Design and layout: Andrey Brem

Publisher LAT & K
Leontovicha Str., 9, Suite 18, Kyiv, Ukraine, 01054
Enlisting the subject of publishing in the State Register
publishers, manufacturers, and distributors
of publishing products.
DK #181 from 15.09.2000
Tel./fax: +38 044 2350009
Cell: +38 050 3102204
E-mail: lk@ukr.net

Circulation 200 copies

9786177824434